1.3 Exercises

1. Describe three everyday situations that involve arrangements of some set of objects.

2. Describe in your own words what a '2-arrangement of 3 objects' means; what an 's-arrangement of t objects' means.

3. List all the 2-arrangements of the symbols (+ − ×). List all the 3-arrangements.

4. Use a tree diagram to list all the 4-arrangements of the letters A, B, C, D.

5. Calculate.
   a) $P(2, 2)$
   b) $P(3, 2)$
   c) $P(4, 2)$
   d) $P(20, 2)$
   e) $P(100, 2)$
   f) $P(n, 2)$
   g) $P(3, 3)$
   h) $P(4, 3)$
   i) $P(5, 3)$
   j) $P(30, 3)$
   k) $P(5, 5)$
   l) $P(4, 5)$

6. Which is largest, $P(20, 1)$, $P(20, 10)$ or $P(20, 20)$?

7. Which is smallest, $P(7, 3)$, $P(4, 3)$ or $P(10, 3)$?

8. Arrange in order from smallest to largest, $P(100, 1)$, $P(10, 2)$, $P(7, 3)$, $P(5, 4)$.

9. Determine the values of $n$ and $r$ for each of the following to be true.
   a) $P(n, r) = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
   b) $P(n, r) = 8 \times 7 \times 6$
   c) $P(n, r) = 510 \times 509 \times 508 \times 507$
   d) $P(n, r) = (a(a - 1)(a - 2)$
   e) $P(n, r) = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3$
   f) $P(n, r) = 7.225$

10. Students are asked to solve the quadratic equation $ax^2 + bx + c = 0$ where the coefficients $a$, $b$ and $c$ can be any value 1, 2, 3, 4 or 5, no two the same. How many different quadratic equations would the students have to solve?

11. Consider the letters of the word HEXAGON.
   a) In how many ways can the letters of the word HEXAGON be arranged amongst themselves?
   b) In how many ways can the vowels of HEXAGON be arranged amongst themselves?
   c) How many arrangements of the letters of HEXAGON begin with an H?
   d) How many arrangements of letters of HEXAGON begin with a vowel? with a consonant?
   e) How are the two answers in part d) related?

12. A computer program is written that generates every arrangement of the letters AEIOLNPRTSY. The program checks each arrangement against a comprehensive set of English words in its memory in an attempt to find any words that have these eleven letters. If each search takes 0.01 s, how much computer time will be required to make a complete search of all arrangements? (Can you make up an eleven letter word from these letters?)

13. a) In how many orders can eight horses finish a race, with no ties in any position?
   b) In how many ways can these horses place 1st, 2nd and 3rd, that is, win, place and show?

14. a) Determine the number of ways five people can occupy five seats.
   b) Determine the number of ways five people can occupy seven seats.

15. You are given the word FIELD.
   a) How many 5-arrangements of these letters are possible?
   b) How many 5-arrangements start with the letter F?
   c) How many 5-arrangements begin with a vowel?
   d) How many 5-arrangements begin with a consonant?
   e) How many 5-arrangements begin and end with a vowel?
   f) How many 5-arrangements begin and end with a consonant?