**Direct and Indirect Reasoning/Complement**

Consider the following Venn diagram.

![Venn Diagram](image)

S = {Universal set}

A = {all elements satisfying some classification described by A}

A' = {all elements that are in the Universal set but are NOT in A}

*** Note: We often refer to A' as the complement of A. ***

The three sets are related by the following equation:

\[ n(A) + n(A') = n(S) \]

\[ n(A) = n(S) - n(A') \]

**Example 1**

Three dice of different colours are tossed one after another.

a) In how many ways can the dice be tossed?

\[ \frac{6 \times 6 \times 6}{D_1 \ D_2 \ D_3} = 216 \]

b) In how many ways can the dice be tossed such that all three resulting faces are different?

\[ \frac{6 \times 5 \times 4}{D_1 \ D_2 \ D_3} = 120 \]

c) In how many ways can the dice be tossed such that at least two of the faces have the same number?

\[ \text{Indirect Reasoning} \]

\[ \# \text{ of ways at least two faces the same} = \text{total outcomes} - \# \text{ of ways where all 3 faces are different} \]

\[ = 216 - 120 \]

\[ = 96 \]
Example 2

In how many ways can six people be selected from a group that consists of four adults and eight children if the group must contain at least one adult.

Strategy 1 (Use Direct Reasoning – Add up all possible cases)

\[
\text{total ways to have at least one adult} = \binom{10}{5} + 2\binom{9}{4} + 3\binom{8}{3} + 4\binom{7}{2} + 5\binom{6}{1} + 6\binom{5}{0}
\]
\[
= (\frac{8!}{5!3!}) + (\frac{8!}{4!4!}) + (\frac{8!}{3!5!}) + (\frac{8!}{2!6!}) + (\frac{8!}{1!7!}) + (\frac{8!}{0!8!})
\]
\[
= 224 + 70 + 56 + 28 + 8 + 1
\]
\[
= 385
\]

Strategy 2 (Use Indirect Reasoning – Consider all cases then remove cases that do not satisfy the requested criteria)

\[
\text{total ways to have at least one adult} = \text{total ways} - \text{# of ways to have zero adults}
\]
\[
= \binom{12}{6} - \binom{4}{0}\binom{8}{6}
\]
\[
= \frac{12!}{6!6!} - \frac{4!}{0!4!}\frac{8!}{6!2!}
\]
\[
= 924 - 28
\]
\[
= 896
\]