**Arrangements and Introduction to Factorials**

**Example 1**

How many three-letter "words" can be formed using the scrabble tiles A, E, and I?

Method 1 (Use Tree Diagram)

```
   E
  / \ (AEI)
 A   I
 /   /
E   I
   /
I   E
```

6 words.

Method 2 (Use the Product Rule/Spaces)

\[
\frac{3 \times 2 \times 1}{1st \quad 2nd \quad 3rd} = 6 \text{ words}
\]

**Example 2**

How many three-letter words can be formed using 7 distinct letters?

\[
\frac{7 \times 6 \times 5}{1st \quad 2nd \quad 3rd} = 210 \text{ words}
\]

**Example 3**

How many seven-letter words can be formed using 7 distinct letters?

\[
\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1st \quad 2nd \quad 3rd \quad 4th \quad 5th \quad 6th \quad 7th} = 5040 \text{ words}
\]

\[7! \text{ reads } 7 \text{ factorial} \]
Factorial Notation

\[ n! = n \times (n - 1) \times (n - 2) \times (n - 3) \ldots 2 \times 1 \]

Ex: \[ 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \]

Example 4

Evaluate the following.

a) \( 4! \)
\[ = 4 \times 3 \times 2 \times 1 \]
\[ = 24 \]

b) \( 8! \)
\[ = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \]
\[ = 40320 \]

Example 5

Evaluate the following.

a) \( \frac{7!}{5!} \)
\[ = \frac{7 \times 6 \times 5!}{5!} \]
\[ = 7 \times 6 \]
\[ = 42 \]

b) \( \frac{10!}{6!} \)
\[ = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!} \]
\[ = 5040 \]

c) \( \frac{100!}{98!} \)
\[ = \frac{100 \times 99 \times 98!}{98!} \]
\[ = 100 \times 99 \]
\[ = 9900 \]

Example 6

Consider the batting order of 9 baseball players (including the pitcher).

a) How many different batting orders are possible?
\[ \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{9!} = 9! = 362880 \]

b) How many different batting orders are possible if the pitcher bats last?
\[ \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1}{8!} \]
\[ = 8! = 40320 \]

Pitcher

c) How many different batting orders are possible if the pitcher bats last and your homerun hitter (slugger) hits 4th?
\[ \frac{7 \times 6 \times 5 \times 1 \times 4 \times 3 \times 2 \times 1 \times 1}{7!} \]
\[ = 7! = 5040 \]