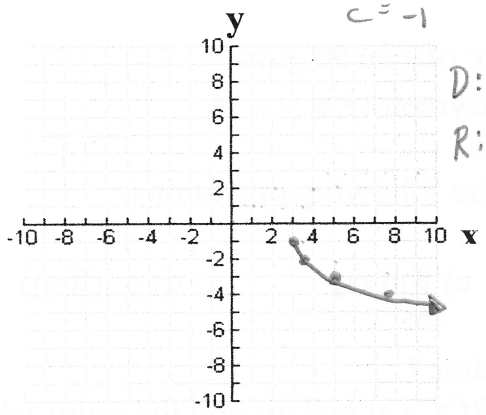
Unit 1: Properties of Functions and Rates of Change

Practice

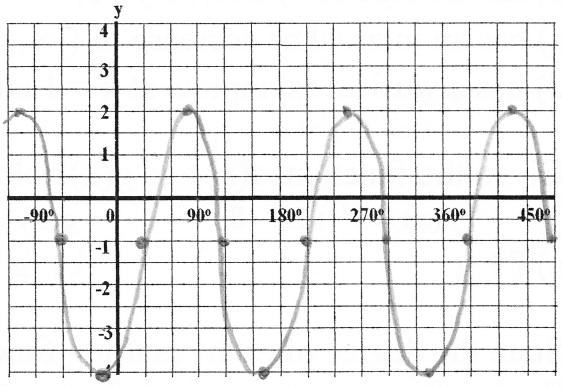
1. List the values of for the constants k, d, a and c then graph the function and state the domain and range.

a) $y = -\sqrt{2x-6} - 1$



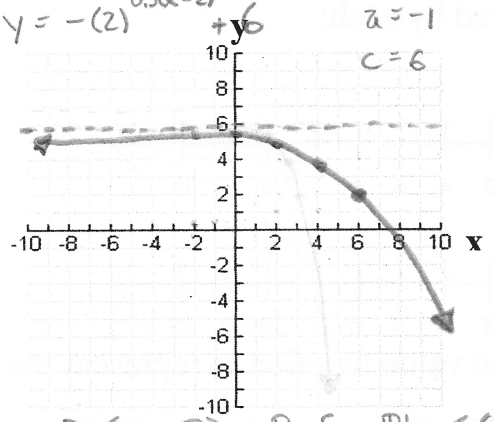
x	y
0	0
1	1
4	2
9	3

b) $y = -3\sin(2\theta + 120) - 1$



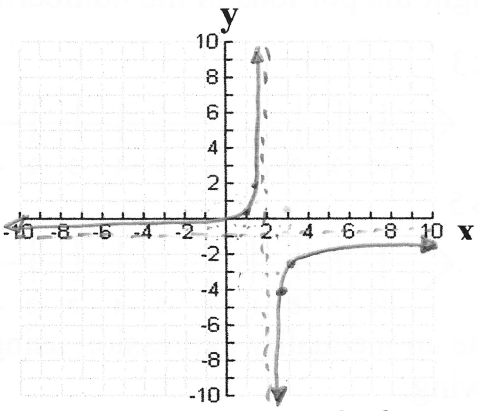
θ	y
0	0
90	1
180	0
270	-1
360	0

c) $y = -2^{0.5x-1} + 6$



x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

d) $y = \frac{3}{-2x+4} - 1$



x	y
-2	-1/2
-1	-1
0	undefined
1	1
2	1/2

2. Determine if the following functions are even, odd or neither.

a) $y = 3x^2 - 8$ b) $f(x) = x^3 - 2x^2$ c) $y = 2\cos[\theta - 90^\circ]$

3. An object is thrown in the air such that its height is modelled by:

$$H(t) = -5t^2 + 40t$$

Determine the rate of change of the height when t = 5 seconds.

4. Determine the inverse of the following functions:

a) $y = \frac{5}{x+2}$ b) $y = -\sqrt{x+2} + 4$ c) $y = (x-1)^2 + 3$

1. On grid.

2a) $f(x) = 3x^2 - 8$

$f(-x) = 3(-x)^2 - 8$

$= 3x^2 - 8$

$f(x) = f(-x)$

$\therefore$ even

b) $f(x) = x^3 - 2x^2$

$f(-x) = (-x)^3 - 2(-x)^2$

$= -x^3 - 2x^2$

$-f(x) = -(x^3 - 2x^2)$

$= -x^3 + 2x^2$

$f(x) \neq f(-x)$

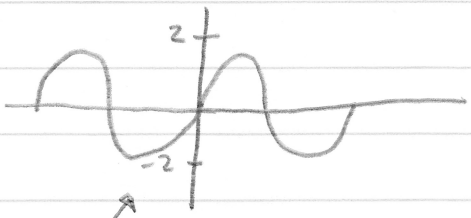
$f(-x) \neq -f(x)$

$\therefore$ Neither

c) $y = 2\cos[\theta - 90°]$

$d = 90°$

$a = 2$



rotational symmetry

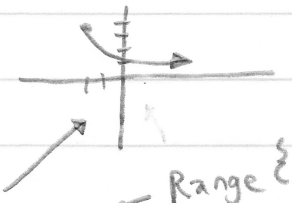
$\therefore$ Odd

3. $IROC \cong \dfrac{H(a+h) - H(a)}{h}$

$\cong \dfrac{H(5.01) - H(5)}{0.01}$

$\cong \dfrac{74.8995 - 75}{0.01}$

$\cong -10.05 m/s$



4.a) $y = \dfrac{5}{x+2}$

$\dfrac{x}{1} = \dfrac{5}{y+2}$

$\dfrac{x(y+2)}{x} = \dfrac{5}{x}$

$y + 2 = \dfrac{5}{x}$

$y = \dfrac{5}{x} - 2$

Inverse

b) $y = -\sqrt{x+2} + 4$

$-\sqrt{y+2} + 4 = x$

$\left(-\sqrt{y+2}\right)^2 = (x-4)^2$

$y + 2 = (x-4)^2$

$y = (x-4)^2 - 2, \; x \leq 4$

Range $\{y \in \mathbb{R} \mid y \leq 4\}$ becomes domain for inverse

c) $y = (x-1)^2 + 3$

$(y-1)^2 + 3 = x$

$\sqrt{(y-1)^2} = \pm\sqrt{x-3}$

$y - 1 = \pm\sqrt{x-3}$

$y = \pm\sqrt{x-3} + 1$

5. $AROC = \frac{\Delta H}{\Delta t}$

$= \frac{H_2 - H_1}{t_2 - t_1}$

$= \frac{H(100) - H(50)}{100 - 50}$

$= \frac{60.19077862 - 37.20944533}{50}$

$\approx +0.46$ m/s

6. Answers vary

a) $y = -(x-3)^2$ or $y = |x-3|^2$ etc

b) $y = 2\sin\theta + 3$ or $y = 2\cos\theta + 3$ etc

c) $y = -2^x - 5$

d) $y = \frac{1}{x-1} - 4$

7. a) $f+g = (x^2+2x-8) + (x-3)$

$= x^2+3x-11$

b) $fg = (x^2+2x-8)(x-3)$

$= x^3-3x^2+2x^2-6x-8x+24$

$= x^3-x^2-14x+24$

c) $f(g(x)) = f(x-3)$

$= (x-3)^2+2(x-3)-8$

$= x^2-6x+9+2x-6-8$

$= x^2-4x-5$

d) $g^{-1}(f(x))$

$= (x^2+2x-8)+3$

$= x^2+2x-5$

$y = x-3$

$x = y-3$

$y = x+3$

$g^{-1}(x) = x+3$

8. a) 2 b) 3 c) 2 d) 2

9. $x=2$ $x=3$ $x=4$

$IROC = \frac{F(2.01)-F(2)}{0.01}$

$= \frac{-20.089699-(-20)}{0.01}$

$= -9$

Negative

$IROC = \frac{F(3.01)-F(3)}{0.01}$

$= \frac{-24.999399-(-25)}{0.01}$

≈ 0.0006

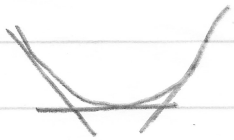
Zero

$IROC = \frac{F(4.01)-F(4)}{0.01}$

$= \frac{-17.849099-(-18)}{0.01}$

≈ 15

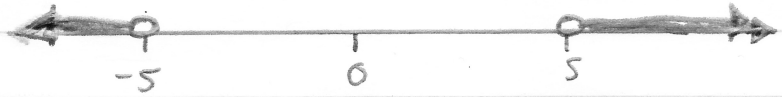
Positive



→ At $x=3$, the stationary point is a local minimum.

10. a) $|x| \leq 3$

b) $|x| > 5$



11. a) $|x| < 5$ b) $|x| \geq 7$

12. a) $y = 2^x - 3$

x-int ($y=0$)

$0 = 2^x - 3$

$2^x = 3$

$\log 2^x = \log 3$

$\frac{x \log 2}{\log 2} = \frac{\log 3}{\log 2}$

x-int ≈ 1.58

y-int ($x=0$)

$y = 2^0 - 3$

$y = 1 - 3$

y-int $= -2$

b) $y = \frac{4}{x-2} - 1$

x-int ($y=0$)

$0 = \frac{4}{x-2} - 1$

$\frac{1}{1} = \frac{4}{x-2}$ ← cross multiply

$x - 2 = 4$

x-int $= 6$

y-int ($x=0$)

$y = \frac{4}{0-2} - 1$

$y = -2 - 1$

y-int $= -3$