MHF4U Proofs: Practice Sheet

1. $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\csc\theta$

L.S. $= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$

$= \frac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$= \frac{2}{\sin\theta}$

R.S. $= 2\csc\theta$

$= \frac{2}{1}\frac{1}{\sin\theta}$

$= \frac{2}{\sin\theta}$

L.S. = R.S.

QED

2. $\frac{\csc\theta+\cot\theta}{\tan\theta+\sin\theta} = \cot\theta\csc\theta$

L.S. $= \frac{\csc\theta+\cot\theta}{\tan\theta+\sin\theta}$

$= \frac{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta(\cos\theta)}{1(\cos\theta)}}$

$= \frac{1+\cos\theta}{\sin\theta} \div \frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}$

$= \frac{(1+\cos\theta)}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin^2\theta}$

R.S. $= \cot\theta\csc\theta$

$= \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}$

$= \frac{\cos\theta}{\sin^2\theta}$

L.S. = R.S.

QED

3. $\frac{2\tan\theta}{1+\tan^2\theta} = \sin(2\theta)$

L.S. $= \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{\frac{2}{1}\cdot\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{2\sin\theta}{\cos\theta} \div \frac{(\cos^2\theta+\sin^2\theta)}{\cos^2\theta}$ ($\cos^2\theta+\sin^2\theta = 1$)

$= \frac{2\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{1}$

$= 2\sin\theta\cos\theta$

R.S. $= \sin 2\theta$

$= 2\sin\theta\cos\theta$

L.S. = R.S.

QED

4. $\sec\theta - \tan\theta\sin\theta = \frac{1}{\sec\theta}$

L.S. $= \sec\theta - \tan\theta\sin\theta$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{1}$

$= \frac{1-\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta}$

$= \cos\theta$

R.S. $= \frac{1}{\sec\theta}$

$= 1 \div \frac{1}{\cos\theta}$

$= 1\cdot\frac{\cos\theta}{1}$

$= \cos\theta$

L.S. = R.S.

QED

5. $\csc^2\theta\tan^2\theta - 1 = \tan^2\theta$

L.S. $= \csc^2\theta\tan^2\theta - 1$ R.S. $= \tan^2\theta$

$= \frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta} - 1$

$= \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}$ L.S. = R.S.

$= \frac{1-\cos^2\theta}{\cos^2\theta}$ QED

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

6. $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

L.S. $= (\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2$ R.S. $= 2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= \sin^2\theta + \cos^2\theta + \sin^2\theta + \cos^2\theta$

$= 1 + 1$

$= 2$

L.S. = R.S.

QED

7. $\frac{\cos\theta+1}{\sin^3\theta} = \frac{\csc\theta}{1-\cos\theta}$

L.S. $= \frac{\cos\theta+1}{\sin^3\theta}$ R.S. $= \frac{\csc\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{\frac{1}{\sin\theta}(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\frac{1+\cos\theta}{\sin\theta}}{\sin^2\theta}$

$= \frac{1+\cos\theta}{\sin\theta} \div \frac{\sin^2\theta}{1}$

$= \frac{\cos\theta+1}{\sin\theta} \cdot \frac{1}{\sin^2\theta}$

$= \frac{\cos\theta+1}{\sin^3\theta}$

L.S. = R.S.

QED

8. $\frac{\sin\theta+\tan\theta}{1+\sec\theta} = \frac{1}{\csc\theta}$

L.S. $= \frac{\sin\theta + \tan\theta}{1+\sec\theta}$ R.S. $= \frac{1}{\csc\theta}$

$= \frac{\frac{(\cos\theta)\sin\theta}{(\cos\theta)1} + \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}$ $= 1 \div \frac{1}{\sin\theta}$

$= 1 \cdot \frac{\sin\theta}{1}$

$= \sin\theta$

$= \frac{\sin\theta\cos\theta + \sin\theta}{\cos\theta} \div \frac{\cos\theta+1}{\cos\theta}$

$= \frac{\sin\theta(\cos\theta+1)}{\cos\theta} \cdot \frac{\cos\theta}{\cos\theta+1}$

$= \sin\theta$

L.S. = R.S.

QED

9. $1 + \cos^4\theta - \sin^4\theta = 2\cos^2\theta$

L.S. $= 1 + \cos^4\theta - \sin^4\theta$

$= 1 + \cos^4\theta - \sin^2\theta \sin^2\theta$

$= 1 + \cos^4\theta - (1-\cos^2\theta)(1-\cos^2\theta)$

$= 1 + \cos^4\theta - [1 - 2\cos^2\theta + \cos^4\theta]$

$= 1 + \cos^4\theta - 1 + 2\cos^2\theta - \cos^4\theta$

$= 2\cos^2\theta$

R.S. $= 2\cos^2\theta$

L.S. = R.S.

QED

10. $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta - \tan\theta)^2$

L.S. $= \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$

$= \frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$

R.S. $= (\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$= \frac{(1-\sin\theta)^2}{(\cos\theta)^2}$

$= \frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}$

L.S. = R.S.

QED

11. $\frac{\tan\theta}{1+\tan^2\theta} = \sin\theta\cos\theta$

L.S. $= \frac{\tan\theta}{1+\tan^2\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\sin\theta}{\cos\theta} \div \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$ ($\cos^2\theta+\sin^2\theta = 1$)

$= \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos^2\theta}{1}$

$= \sin\theta\cos\theta$

R.S. $= \sin\theta\cos\theta$

L.S. = R.S.

QED

12. $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = 2\csc^2\theta$

L.S. $= \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$

$= \frac{1}{(1-\cos\theta)}\frac{(1+\cos\theta)}{(1+\cos\theta)} + \frac{1}{(1+\cos\theta)}\frac{(1-\cos\theta)}{(1-\cos\theta)}$

$= \frac{1+\cos\theta}{1-\cos^2\theta} + \frac{1-\cos\theta}{1-\cos^2\theta}$

$= \frac{1+\cos\theta}{\sin^2\theta} + \frac{1-\cos\theta}{\sin^2\theta}$

$= \frac{1+\cos\theta+1-\cos\theta}{\sin^2\theta}$

$= \frac{2}{\sin^2\theta}$

R.S. $= 2\csc^2\theta$

$= \frac{2}{1}\frac{1}{\sin^2\theta}$

$= \frac{2}{\sin^2\theta}$

L.S. = R.S.

QED