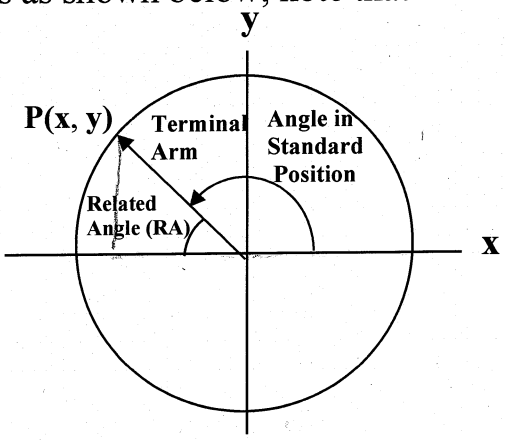
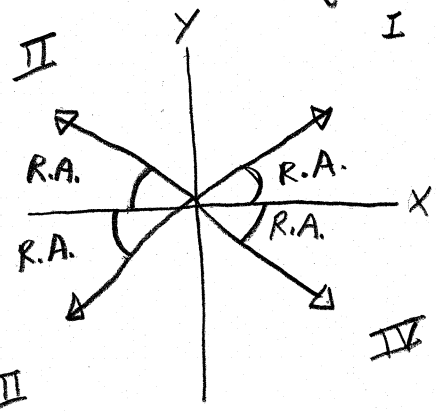
Grade 11 Review – Trigonometry

The relationship between the coordinates of a point on the end of terminal arm, the length of this terminal arm and the angle in standard position can be expressed by three trigonometric functions as shown below; note that $r^2 = x^2 + y^2$



Related Angles



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

SYR CXR TYX

Example 1

Determine the three trigonometric ratios of a terminal arm that extends to the point (-2, 5).

$r^2 = x^2 + y^2$
$r^2 = (-2)^2 + (5)^2$
$\sqrt{r^2} = \sqrt{29}$
$r = \sqrt{29}$

$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$

$\tan\theta = \frac{y}{x} = \frac{5}{-2} = -\frac{5}{2}$

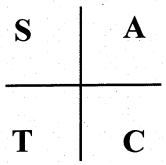
$\sqrt{29}\sqrt{29} = 29^{1/2}29^{1/2} = 29^1 = 29$

X Y

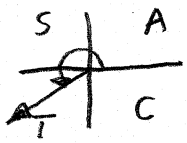
The "CAST Rule" is used to determine the sign (+ or -) when evaluating a trigonometric ratio.

Example 2

Determine the sign of each trigonometric expression below; do not evaluate.

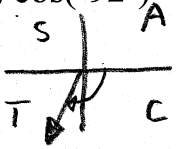


a) sin(210°)



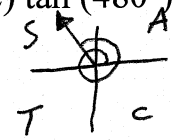
→ negative

b) cos(-92°)



→ negative

c) tan (480°)



→ negative

When an angle is calculated using an inverse trigonometric function, there are an infinite number of solutions; a calculator will only give you one of these solutions. You need to do further analysis using knowledge of the CAST Rule and related angles to find other solutions.

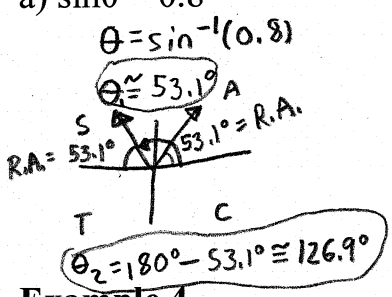
Example 3

Solve the following trigonometric equations given $0^o \leq \theta \leq 360^o$.

a) $\sin\theta = 0.8$

$\theta = \sin^{-1}(0.8)$

$\theta \cong 53.1°$

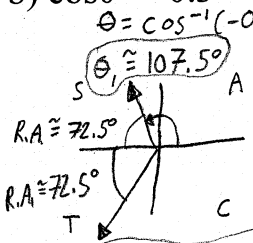


$\theta_2 = 180° - 53.1° \cong 126.9°$

b) $\cos\theta = -0.3$

$\theta = \cos^{-1}(-0.3)$

$\theta_1 \cong 107.5°$

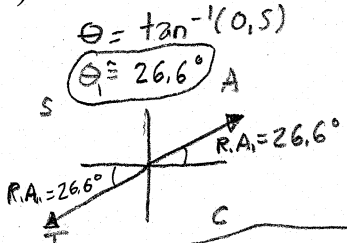


$\theta_2 \cong 180° + 72.5° \cong 252.5°$

c) $\tan\theta = 0.5$

$\theta = \tan^{-1}(0.5)$

$\theta_1 \cong 26.6°$



$\theta_2 = 180° + 26.6° \cong 206.6°$

Example 4

For the previous example, find four other solutions (co-terminal angles) for $\sin\theta = 0.8$ outside of the limits $0^o \leq \theta \leq 360^o$.

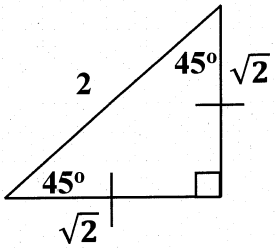
$\theta_3 = 360° + 53.1° \cong 413.1°$

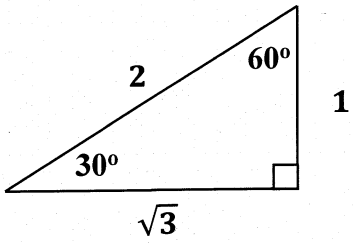
$\theta_4 = -360° + 53.1° \cong -306.9°$

$\theta_5 = 360° + 126.9° \cong 486.9°$

$\theta_6 = -360° + 126.9° \cong -233.1°$

The trigonometric measure for special angles (30°, 45°, and 60°) can be determined by memorizing the following two triangles:





Example 5

SOH CAH TOA

Determine the exact value of each trigonometric expression.

a) $\sin(60°) = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$

b) $\cos(45°) = \frac{adj}{hyp} = \frac{\sqrt{2}}{2}$

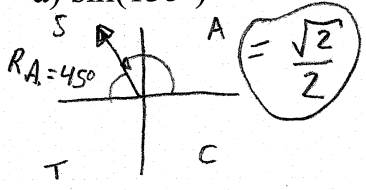
c) $\tan(30°) = \frac{opp}{adj} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

* The trigonometric ratio of any non-acute angle can by determined by evaluating the ratio of its related angle then changing the resulting sign to a negative if required as per the CAST rule.

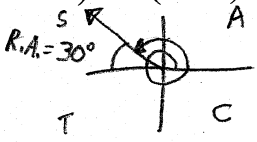
Example 6

Determine the value of each trigonometric expression; be exact if possible.

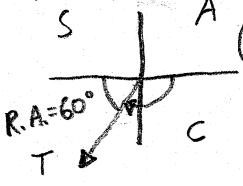
a) $\sin(135°) = \sin(45°) = \frac{\sqrt{2}}{2}$



b) $\cos(510°) = -\cos(30°) = -\frac{\sqrt{3}}{2}$



c) $\tan(-120°) = \tan(60°) = \sqrt{3}$



Each primary trigonometric function has a corresponding reciprocal function as follows.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Example 7

Evaluate each expression.

a) $\csc(60^\circ)$

$= \frac{1}{\sin(60^\circ)}$ $= \frac{1}{\sqrt{3}/2}$ $= 1 \div \frac{\sqrt{3}}{2}$ $= 1 \cdot \frac{2}{\sqrt{3}}$ $= \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $= \frac{2\sqrt{3}}{3}$

b) $\cot(300^\circ)$

$= \frac{1}{\tan(300^\circ)}$ $= \frac{1}{-\tan(60^\circ)}$ $= -\frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$ $= -\frac{\sqrt{3}}{3}$

S A T C R.A.=60°

Example 8

Solve each equation; $0^\circ \le \theta \le 360^\circ$

a) $\csc\theta = 1.5$

$\frac{1}{\sin\theta} = \frac{1.5}{1}$ $\frac{1.5\sin\theta}{1.5} = \frac{1}{1.5}$ $\sin\theta = \frac{1}{1.5}$ $\theta = \sin^{-1}\left(\frac{1}{1.5}\right)$ $\theta_1 \doteq 41.8^\circ$ $\theta_2 \doteq 180^\circ - 41.8^\circ$ $\theta_2 \doteq 138.2^\circ$

S A T C R.A.=41.8° 41.8°=R.A.

b) $\sec\theta = 2$

$\cos\theta = \frac{1}{2}$ $\theta = \cos^{-1}\left(\frac{1}{2}\right)$ $\theta_1 = 60^\circ$ $\theta_2 = 360^\circ - 60^\circ$ $\theta_2 = 300^\circ$

S A T C R.A.=60° R.A.=60°

Practice

1. Determine the three trigonometric ratios for a terminal arm that extends to the point (5, -12) and determine the angle in standard position.
2. Determine the sign of each trigonometric ratio using the CAST rule.
   a) $\sin(800^\circ)$ b) $\cos(-120^\circ)$ c) $\tan(120^\circ)$
3. Solve the following trigonometric equations given $0^\circ \le \theta \le 360^\circ$.
   a) $\sin\theta = -0.5$ b) $\cos\theta = 0.3$ c) $\tan\theta = 1.2$
4. Determine 2 co-terminal angles (one position and one negative) for the angle 125°.
5. Determine the exact value of each trigonometric expression.
   a) $\sin(300^\circ)$ b) $\tan(420^\circ)$
6. Evaluate each expression.
   a) $\cot(-30^\circ)$ b) $\sec(-585^\circ)$
7. Solve each equation; $0^\circ \le \theta \le 360^\circ$
   a) $\sec\theta = 0.8$ b) $\cot\theta = 1$
8. Use the formulas SYR CXR TXY to verify/prove the following identities:
   a) $\sin^2\theta + \cos^2\theta = 1$ b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$
9. Use your understanding of the CAST rule and related angles to explain why the following is true:
   $\cos(\theta) = \cos(-\theta)$

Solutions

1.a) $\sin\theta = -12/13$, $\cos\theta = 5/13$, $\tan\theta = -12/5$, $\theta = 292.6^\circ$, 2.a) positive, b) negative, c) negative
3.a) $\theta = 210^\circ$ or 330° b) $\theta = 72.5^\circ$ or 287.5° c) 50.2° or 130.2° 4. 485° or -235°
5. a) $-\frac{\sqrt{3}}{2}$, b) $\sqrt{3}$, 6. a) $-\sqrt{3}$, b) $-\sqrt{2}$, 7. a) No solution, b) $\theta = 45^\circ$ or 225°
8. Change $\sin^2\theta = \frac{y^2}{r^2}$, etc… then replace $y^2 + x^2$ with r^2.
9. Same related angle (and values for x and r) by rotating clockwise or anticlockwise from the initial arm.