Trigonometric Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identity

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

The Pythagorean Identity (and rearranged)

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

Other Pythagorean Identities

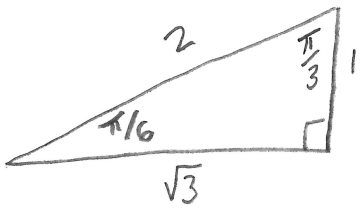
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

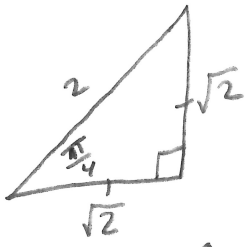
Compound Formulas

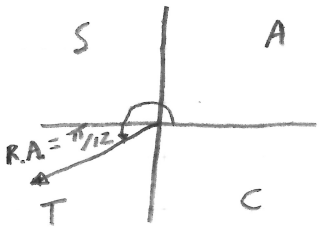
$$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$$
$$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$$
$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a\tan b}$$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$







2a) $\cos\left(\frac{5\pi}{12}\right)$ ← 75°

$= \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

b) $\sin\left(\frac{13\pi}{12}\right)$ ← 195°

$= -\sin\left(\frac{\pi}{12}\right)$

$= -\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$= -\left[\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)\right]$

$= -\left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)\right]$

$= -\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$

$= \boxed{\frac{-\sqrt{6}+\sqrt{2}}{4}}$

3a) $\tan\left(\frac{2\pi}{3}\right)$

$= \tan\left[2\left(\frac{\pi}{3}\right)\right]$

$= \frac{2\tan(\pi/3)}{1-\tan^2(\pi/3)}$

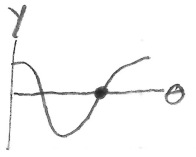
$= \frac{2\sqrt{3}}{1-(\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{1-3}$

$= \frac{2\sqrt{3}}{-2}$

$= \boxed{-\sqrt{3}}$

b) $\cos\left(\frac{3\pi}{2}\right)$



$\frac{3\pi}{2} \div \frac{2}{1} = \frac{3\pi}{2}\cdot\frac{1}{2} = \frac{3\pi}{4}$

$= \cos\left[2\left(\frac{3\pi}{4}\right)\right]$

$= 2\cos^2\left(\frac{3\pi}{4}\right) - 1$

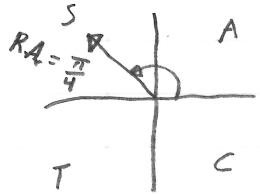
$= 2\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right) - 1$

$= 2\left[-\cos\left(\frac{\pi}{4}\right)\right]\left[-\cos\left(\frac{\pi}{4}\right)\right] - 1$

$= \frac{2}{1}\left[-\frac{\sqrt{2}}{2}\right]\left[-\frac{\sqrt{2}}{2}\right] - 1$

$= \frac{4}{4} - 1$

$= \boxed{\emptyset}$



4a) $\frac{1+\sin\theta}{1-\sin\theta} = \left[\tan\theta + \frac{1}{\cos\theta}\right]^2$

$L.S. = \frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$

$R.S. = \left[\tan\theta + \frac{1}{\cos\theta}\right]^2$

$= \left[\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right]^2$

$= \left[\frac{\sin\theta + 1}{\cos\theta}\right]^2$

$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$

$L.S. = R.S.$

QED

b) $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$

$L.S. = \frac{1-2\cos^2\theta}{\sin\theta\cos\theta}$

$R.S. = \tan\theta - \cot\theta$

$= \frac{\sin\theta}{\cos\theta}\frac{\sin\theta}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\frac{\cos\theta}{\cos\theta}$

$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{(1-\cos^2\theta) - \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1-2\cos^2\theta}{\sin\theta\cos\theta}$

$L.S. = R.S.$

QED

5. $\sin\theta$ ↓ $\sin\left(\frac{\pi}{8}\right) = ?$ given $\cos(2\theta)$ ↓ $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

set $\theta = \frac{\pi}{8}$

$$\cos\left[2\left(\frac{\pi}{8}\right)\right] = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\cos\left(\frac{\pi}{4}\right) = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

$$\frac{1}{2}\left[2\sin^2\left(\frac{\pi}{8}\right)\right] = \left[\frac{1 - \cos\left(\frac{\pi}{4}\right)}{1}\right]\frac{1}{2}$$

$$\sin^2\left(\frac{\pi}{8}\right) = \frac{1 - \sqrt{2}/2}{2}$$

$$\sin^2\left(\frac{\pi}{8}\right) = \frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{2/1}$$

$$\sin^2\left(\frac{\pi}{8}\right) = \frac{2 - \sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\sqrt{\sin^2\left(\frac{\pi}{8}\right)} = \pm\sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$\sin\left(\frac{\pi}{8}\right) = \pm\frac{\sqrt{2 - \sqrt{2}}}{\sqrt{4}}$$

$$\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

← must be positive since $\pi/8$ puts terminal arm in quadrant 1.

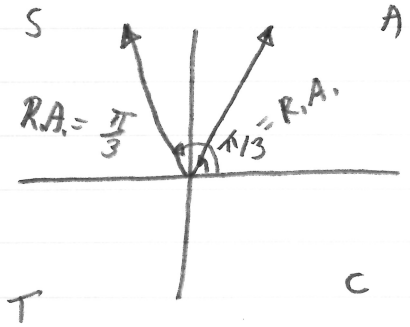
$0 \le \theta \le 2\pi$

6a) $2\sin\theta - \sqrt{3} = 0$

$\frac{2\sin\theta}{2} = \frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$R.A. = \frac{\pi}{3}$



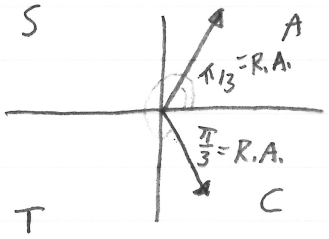
$\theta_1 = \pi/3 \qquad \theta_2 = \frac{2\pi}{3}$

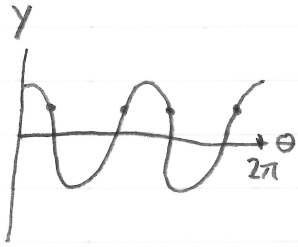
b) $2\cos(2\theta) - 1 = 0$

$\frac{2\cos(2\theta)}{2} = \frac{1}{2}$

$\cos(2\theta) = \frac{1}{2}$

$R.A. = \pi/3$





$\frac{1}{2}(2\theta) = (\frac{\pi}{3})\frac{1}{2}$ $\quad \theta_1 = \frac{\pi}{6}$

$\frac{1}{2}(2\theta) = (\frac{5\pi}{3})\frac{1}{2}$ $\quad \theta_2 = \frac{5\pi}{6}$

$\frac{1}{2}(2\theta) = (\frac{7\pi}{3})\frac{1}{2}$ $\quad \theta_3 = \frac{7\pi}{6}$

$\frac{1}{2}(2\theta) = (\frac{11\pi}{3})\frac{1}{2}$ $\quad \theta_4 = \frac{11\pi}{6}$

Alternatively...

$2\cos(2\theta) - 1 = 0$

$\cos(2\theta) = \frac{1}{2}$

$2\cos^2\theta - 1 = \frac{1}{2}$

$2\cos^2\theta = \frac{1}{2} + \frac{2}{2}$

$\frac{1}{2}(2\cos^2\theta) = (\frac{3}{2})\frac{1}{2}$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{3}{4}}$

$\cos\theta = \frac{\sqrt{3}}{2}$ or $\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta_1 = \frac{\pi}{6}$ $\quad \theta_3 = \frac{5\pi}{6}$

$\theta_2 = \frac{11\pi}{6}$ $\quad \theta_4 = \frac{7\pi}{6}$