Homework: Complete Worksheet + pg 35 # 1ace, 2a, 3, 4ace, 5ace, 8, 9af, 10ab, 11, 13, (15)

Review Grade 11 – Part 1: Transformation of Functions

If f(x) represents some parent function, transformations can be used to create a new function:

inside (horizontal, think opposite)

$$y = af[k(x-d)] + c$$

outside (vertical, follow intuition)

The graph of this new function will have similarities to the original parent function f(x), but each point will have been transformed as a result of the constants k, d, a, and c. The effect of each of these constants is the following:

k --> horizontally expands/compresses by a factor of $\frac{1}{|k|}$. If 'k' is negative then the point is subsequently reflected about the y-axis.
d --> horizontally shifts the point 'd' units to the right.
a --> vertically expands/compresses by a factor of |a|. If 'a' is negative, then the point is subsequently reflected about the x-axis.
c --> vertically shifts the point up 'c' units.

A few key things to note

- Constants that are inside the function (k and d) transform the function horizontally; the effect of these constants is opposite of our intuition.
- Constants that are outside the function (a and c) transform the function vertically.
- Constants that operate as multipliers (k and a) represent expansion/compressions.
- Constants that operate as addition or subtraction (d and c) represent shifts.

Order
→ k before d
→ a before c

k, d, a, c
k, a, d, c

Example 1

Describe the transformations in order for the following functions.

a) y = -4f(-0.5(x - 1)) + 3

	Constant	Transformation
h	k = $-\frac{1}{2}$	→ horizontal expansion by a factor of 2 → reflection about the y-axis
h	d = 1	→ horizontal shift right 1 unit
v	a = -4	→ vertical expansion by a factor of 4. → reflection about the x-axis
v	c = 3	→ vertical shift up 3 units

b) Given the parent function $f(x) = \sin(\theta)$,

$$y = 3\sin(2\theta - 120^\circ) - 1$$

$y = 3\sin[2(\theta - 60^\circ)] - 1$

	Constant	Transformation
h	k = 2	→ horizontal compression by a factor of $\frac{1}{2}$
h	d = 60°	→ phase shift right 60°
v	a = 3	→ vertically expand by a factor of 3.
v	c = -1	→ shift down 1 unit.

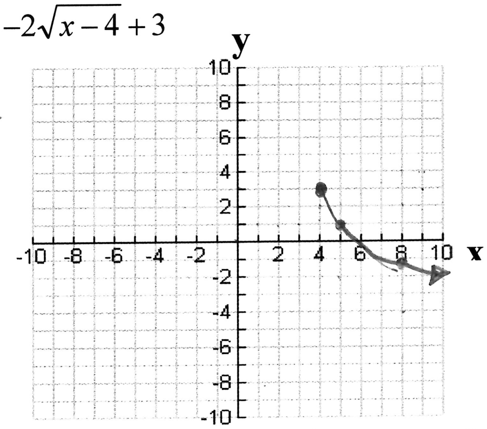
Example 2

Give the following table of values for parent functions create a graph of the transformed function and state the domain and range.

x	$y=\sqrt{x}$
0	0
1	1
4	2
9	3

x	$y=\lvert x\rvert$
-2	2
-1	1
0	0
1	1
2	2

a) $y=-2\sqrt{x-4}+3$



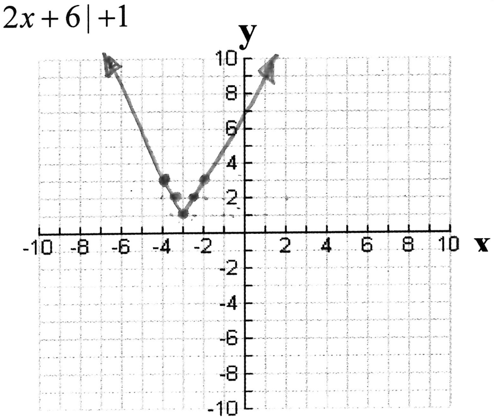
h
~~**k** = 1~~
d = 4

v
a = -2
c = 3

Domain: $\{x\in\mathbb{R}\mid x\geq 4\}$ **Range:** $\{y\in\mathbb{R}\mid y\leq 3\}$

$y=|2(x+3)|+1$

b) $y=|2x+6|+1$



h
k = 2
d = -3

v
~~**a** = 1~~
c = 1

Domain: $\{x\in\mathbb{R}\}$ **Range:** $\{y\in\mathbb{R}\mid y\geq 1\}$

Mapping Function

Another way to transform a function is to use a mapping statement as follows:

Parent function — Transformed Function

(x, y) --------> $\left(\dfrac{x}{k}+d,\ ay+c\right)$

Example

Example 3

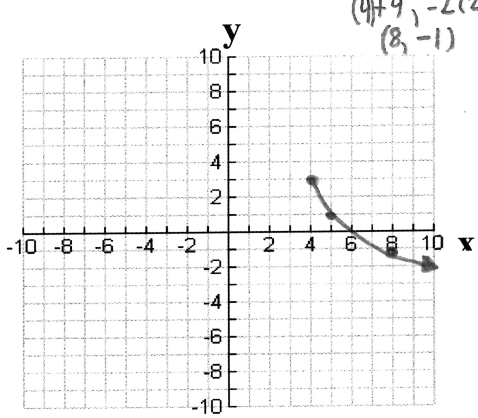
Create a mapping function and use it to recreate the graphs above.

a) $y=-2\sqrt{x-4}+3$ **(x, y) -->** $(x+4, -2y+3)$

Example (4,2)

$(4+4,\ -2(2)+3)$

$(8, -1)$



k = 1

d = 4

a = -2

c = 3

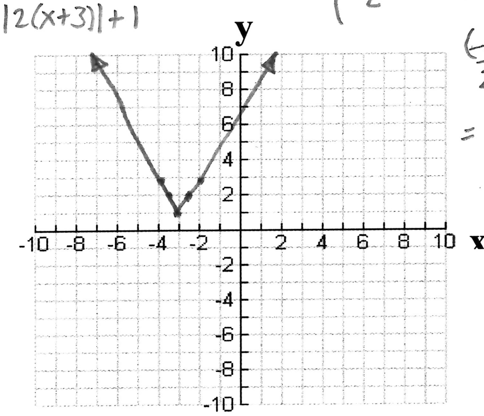
b) $y=|2x+6|+1$ **(x, y) -->** $\left(\dfrac{x}{2}-3,\ y+1\right)$

Example (-2,2)

$y=|2(x+3)|+1$

$\dfrac{(-2)}{2}-3,\ (2)+1$

$=-4, 3$



k = 2

d = -3

a = 1

c = 1

Practice

Given a table of values for the following parent functions, graph the following and state the domain and range:

$y = \sqrt{x}$ $\qquad$ $y = |x|$ $\qquad$ $y = \dfrac{1}{x}$

x	y
0	0
1	1
4	2
9	3

x	y
-2	2
-1	1
0	0
1	1
2	2

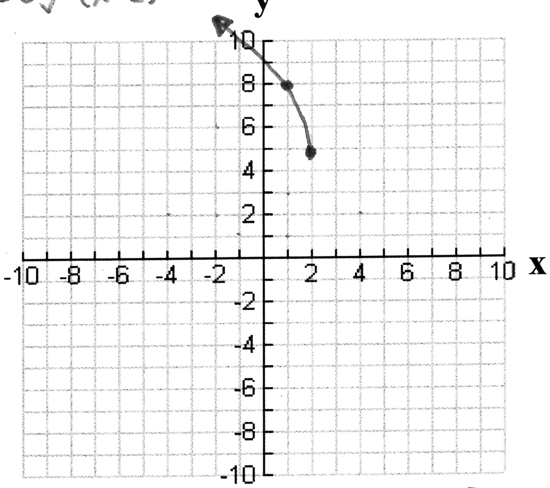
x	y
-2	-0.5
-1	-1
0	DNE
1	1
2	0.5

a) $y = 3\sqrt{-x+2} + 5$

$y = 3\sqrt{-(x-2)} + 5$

h: k = -1, d = 2

v: a = 3, c = 5



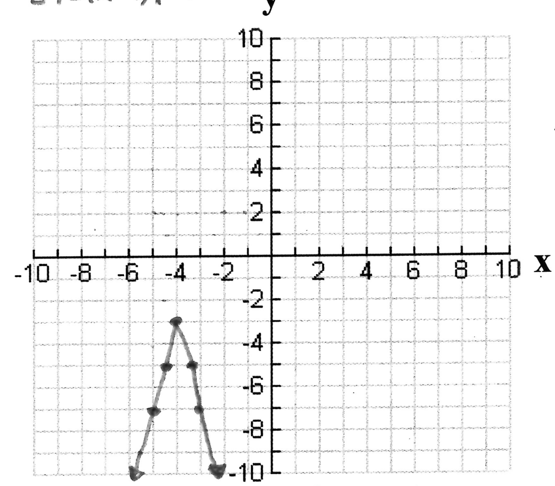
Domain: $\{x \in \mathbb{R} \mid x \leq 2\}$ Range: $\{y \in \mathbb{R} \mid y \geq 5\}$

b) $y = -2|2x+8| - 3$

$y = -2|2(x+4)| - 3$

h: k = 2, d = -4

v: a = -2, c = -3



Domain: $\{x \in \mathbb{R}\}$ Range: $\{y \in \mathbb{R} \mid y \leq -3\}$

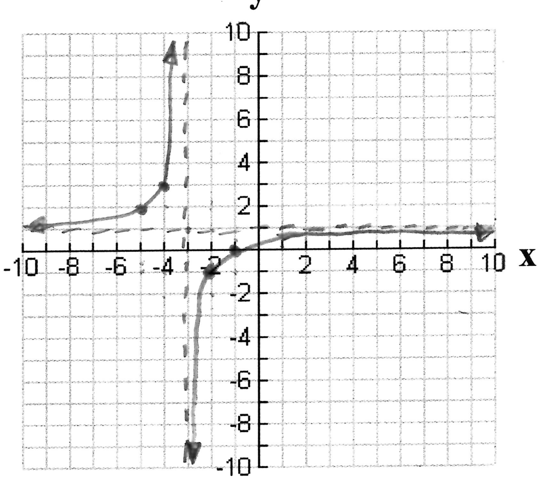
c) $y = -\dfrac{2}{x+3} + 1 \longrightarrow y = -2\dfrac{1}{x+3} + 1$

~~k = 1~~

d = -3

a = -2

c = 1

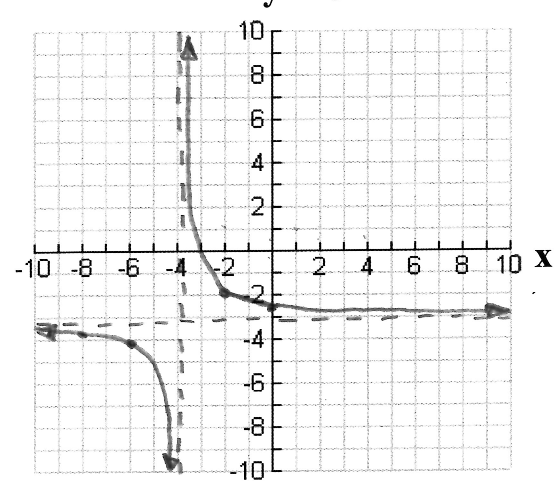


Domain: $\{x \in \mathbb{R} \mid x \neq -3\}$ Range: $\{y \in \mathbb{R} \mid y \neq 1\}$

d) $y = \dfrac{1}{0.5x+2} - 3 \longrightarrow y = \dfrac{1}{\frac{1}{2}(x+4)} - 3$

h: k = $\frac{1}{2}$, d = -4

v: ~~a = 1~~, c = -3



Domain: $\{x \in \mathbb{R} \mid x \neq -4\}$ Range: $\{y \in \mathbb{R} \mid y \neq -3\}$