Homework: Complete Practice Sheet + pg 343 #1ac, 4ac, 5ac, 6ac, 9, *14

Transformations of Sinusoidals: Part 2

Recall: The five points for the parent sinusoidal functions below.

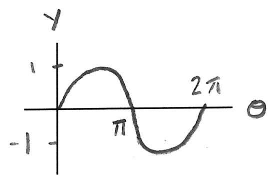
θ	$y = \sin\theta$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

θ	$y = \cos\theta$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

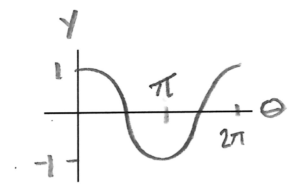
Exercise

Sketch one cycle of each of the following functions.

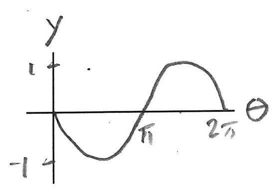
$y = \sin\theta$



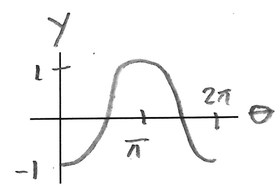
$y = \cos\theta$



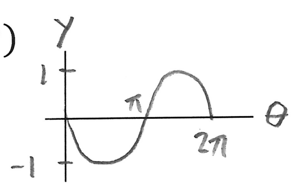
$y = -\sin\theta$



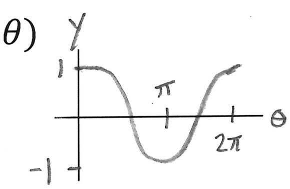
$y = -\cos\theta$



$y = \sin(-\theta)$



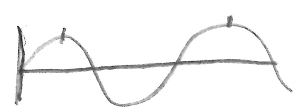
$y = \cos(-\theta)$



The constant k is related to the period, T, by the following equations:

$$\boxed{k = \frac{2\pi}{T}} \quad \text{or} \quad \boxed{T = \frac{2\pi}{|k|}}$$

→ small period
→ large freq.



→ large period
→ small freq.

- When k is large the period is small and the frequency is large.
- When k is small the period is large and the frequency is small.

$\frac{3\pi}{8} \div \frac{1}{4} = \frac{3\pi}{8} \cdot \frac{4}{1} \to \frac{12\pi}{8} = \frac{3\pi}{2}$ $\frac{\pi}{2} \div \frac{2\pi}{1} = \frac{\pi}{2} \cdot \frac{1}{2\pi} = \frac{1}{4}$

Example 1

$\frac{\pi}{4}$

For each sinusoidal function listed below, determine the period and phase.

a) $y = \sin(4\theta - \pi)$

$y = \sin\left[4\left(\theta - \frac{\pi}{4}\right)\right]$

$T = \frac{2\pi}{|k|} = \frac{2\pi}{|4|} = \frac{\pi}{2}$ Phase (d) $= \frac{\pi}{4}$

b) $y = \cos\left(0.25\theta + \frac{3\pi}{8}\right)$

$y = \cos\left[0.25\left(\theta + \frac{3\pi}{2}\right)\right]$

$T = \frac{2\pi}{|k|} = \frac{2\pi}{|0.25|} = 8\pi$ Phase $= -\frac{3\pi}{2}$

c) $y = \cos\left(2\pi\theta + \frac{\pi}{2}\right)$

$y = \cos\left[2\pi\left(\theta + \frac{1}{4}\right)\right]$

$T = \frac{2\pi}{|k|} = \frac{2\pi}{|2\pi|} = 1$ Phase $= -\frac{1}{4}$

Example 2

Determine the k value for the following periods.

a) T = 12

$k = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$

b) T = 20 seconds

$k = \frac{2\pi}{T} = \frac{2\pi}{20} = \frac{\pi}{10}$

Example 3

Create a sinusoidal function that has a phase shift of $\frac{\pi}{3}$ radians, an amplitude of ~~5~~ 5, an axis of equilibrium at $y = -3$ and a period of π radians.

$k = \frac{2\pi}{T} = \frac{2\pi}{\pi}$ $k = 2$

$d = \frac{\pi}{3}$

$a = -5$

$c = -3$

$y = a\cos[k(\theta - d)] + c$

$y = -5\cos\left[2\left(\theta - \frac{\pi}{3}\right)\right] - 3$

Example 4

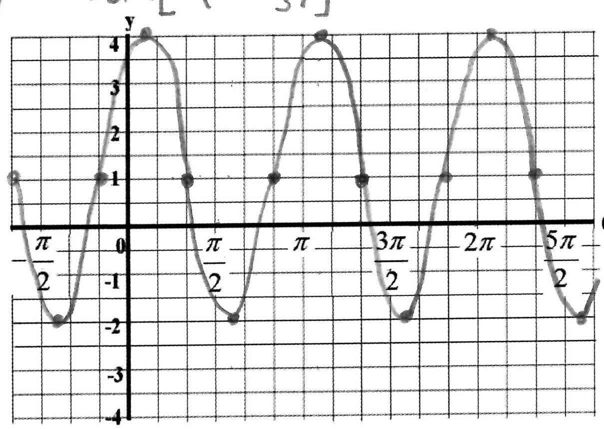
Graph the first function using the five points system and the second function using the box method. Note if k is negative when using the box method, draw a sinusoidal that has been mirror reflected about the y-axis.

a) $y = -3\sin\left(2\theta - \frac{2\pi}{3}\right) + 1$

$y = -3\sin\left[2\left(\theta - \frac{\pi}{3}\right)\right] + 1$

h $\left[\begin{array}{l} k = 2 \\ d = \frac{\pi}{3} \end{array}\right.$

v $\left[\begin{array}{l} a = -3 \\ c = 1 \end{array}\right.$



b) $y = -2\cos(-3\theta + 3\pi) - 2$

$y = -2\cos[-3(\theta - \pi)] - 2$

$T = \frac{2\pi}{|k|} = \frac{2\pi}{|-3|} = \frac{2\pi}{3}$

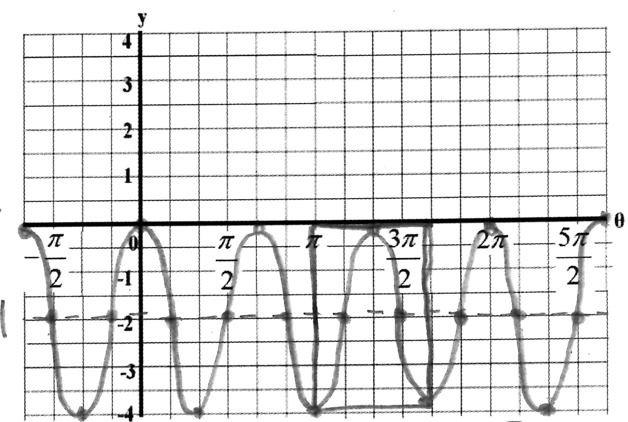
$k = -3$

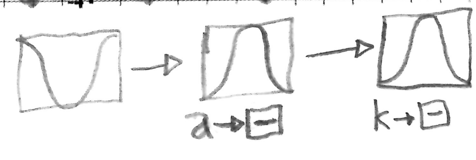
$d = \pi$

$a = -2$

$c = -2$

amp $= |a| = |-2| = 2$





Practice

Graph each function using a method of your choosing.

a) $y = \sin\left(2\theta + \dfrac{\pi}{3}\right)$

$y = \sin\left[2\left(\theta + \frac{\pi}{6}\right)\right] + 0$

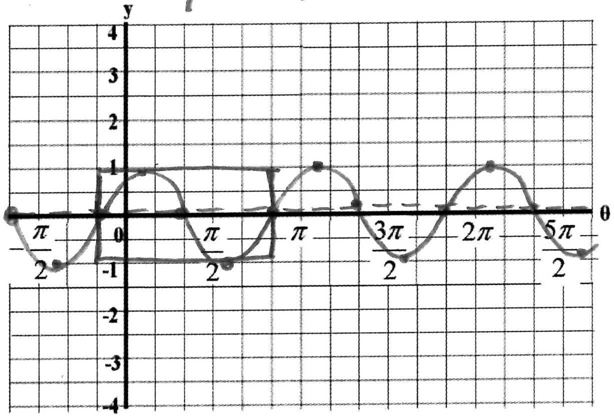
$k = 2$

$d = -\frac{\pi}{6}$

$a = 1$

$c = 0$

$T = \pi$



b) $y = 2\cos(-3\theta + \dfrac{\pi}{2})$ $\rightarrow y = 2\cos\left[-3\left(\theta - \frac{\pi}{6}\right)\right] + 0$

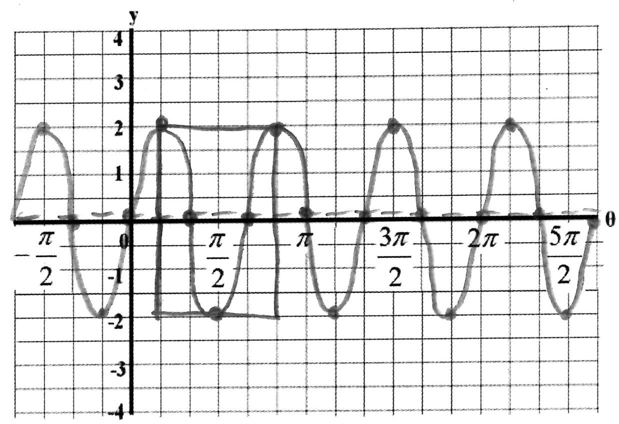
$k = -3$

$d = \frac{\pi}{6}$

$a = 2$

$c = 0$

$T = \frac{2\pi}{3}$



c) $y = 3\sin\left(\theta - \dfrac{\pi}{2}\right) - 1$

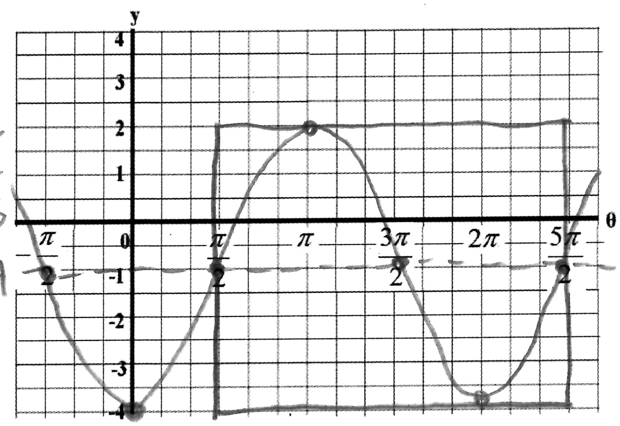
$k = 1$

$d = \frac{\pi}{2}$

$a = 3$

$c = -1$

$T = 2\pi$



d) $y = -2\cos\left(2\theta + \dfrac{2\pi}{3}\right)$ $\rightarrow y = -2\cos\left[2\left(\theta + \frac{\pi}{3}\right)\right] + 0$

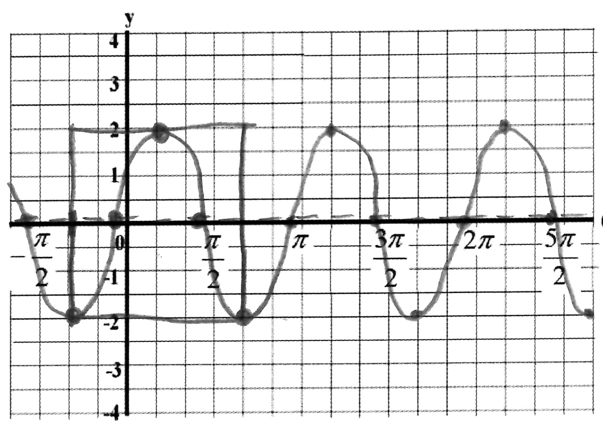
$k = 2$

$d = -\frac{\pi}{3}$

$a = -2$

$c = 0$

$T = \pi$



e) $y = 2\sin(0.75\theta) + 1$

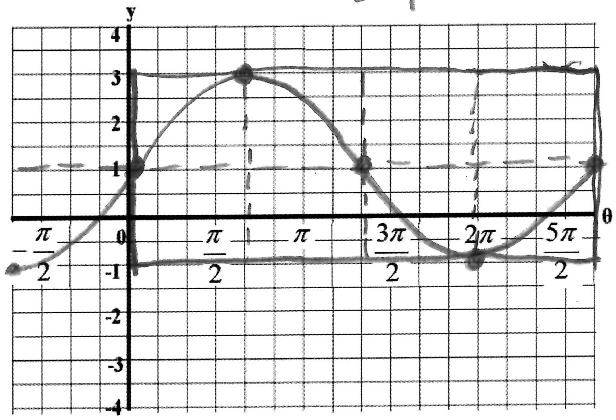
$T = \frac{2\pi}{|3/4|} = \frac{2\pi}{1} \cdot \frac{4}{3} \rightarrow = \frac{8\pi}{3}$

$k = \frac{3}{4}$

$d = 0$

$a = 2$

$c = 1$



f) $y = 3 - \cos\left(-\theta + \dfrac{3\pi}{2}\right)$ $\rightarrow y = -1\cos\left[-1\left(\theta - \frac{3\pi}{2}\right)\right] + 3$

$k = -1$

$d = \frac{3\pi}{2}$

$a = -1$

$c = 3$

$T = 2\pi$