Hmwk: pg 182 #2-4(ace), 5-8

Factoring a Sum and Difference of Cubes

Recall: Difference of Squares

$a^2 - b^2 = (a-b)(a+b)$

Factor the following by recognizing that it is a difference of squares.

* common factor first

a) $x^2 - 9$

$= (x-3)(x+3)$

b) $4x^2 - 81$

$= (2x-9)(2x+9)$

c) $25x^2 - 100y^2$

$= 25(x^2 - 4y^2)$

$= 25(x-2y)(x+2y)$

Note: There is no such thing as factoring a sum of squares.

For a binomial made up of two terms that are each cubes, there is a factoring rule for both sums and differences; see below.

Example 1

Determine the factors of a sum of cubes and difference of cubes by dividing the following. $(a^2-b^2) = (a-b)(a+b)$

a) $(a^3 + b^3) \div (a+b)$

$$
\begin{array}{r}
a^2 - ab + b^2, \ R0 \\
a+b \,\big)\, a^3 + 0a^2b + 0ab^2 + b^3 \\
a^3 + a^2b \\
\hline
-a^2b + 0ab^2 \\
-a^2b - ab^2 \\
\hline
ab^2 + b^3 \\
ab^2 + b^3 \\
\hline
0
\end{array}
$$

b) $(a^3 - b^3) \div (a-b)$

$$
\begin{array}{r}
a^2 + ab + b^2, \ R\emptyset \\
a-b \,\big)\, a^3 + 0a^2b + 0ab^2 - b^3 \\
a^3 - a^2b \\
\hline
a^2b + 0ab^2 \\
a^2b - ab^2 \\
\hline
ab^2 - b^3 \\
ab^2 - b^3 \\
\hline
0
\end{array}
$$

Sum of Cubes	Difference of Cubes
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example 2

Factor the following:

a) $x^3 + 64$

$a = x \quad b = 4$

$= (x+4)(x^2 - 4x + 16)$

$b^2 - 4ac = (-4)^2 - 4(1)(16)$

$= -48$

negative $\therefore$ not factorable

b) $-7x^3 + 56$

$= -7(x^3 - 8)$

$a = x \quad b = 2$

$= -7(x-2)(x^2 + 2x + 4)$

c) $3(x+2)^3 + 3$

$a = (x+2)$
$b = 1$

$= 3[(x+2)^3 + 1]$

$= 3[(x+2)+1][(x+2)^2 - (x+2) + 1]$

$= 3[x+3][x^2 + 4x + 4 - x - 2 + 1]$

$= 3[x+3][x^2 + 3x + 3]$