Stationary Points

Hmwk: pg 111 # 1, 2, 5ac, 6adf, 10, 15(only for x^2)

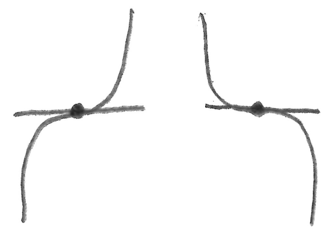
Stationary point – a location on the graph where the instantaneous rate of change is equal to zero.

Stationary points play a significant role in mathematics when we are interested in determining the maximum or minimum value of a function; this topic in mathematics is often referred to as optimization and is discussed in more detail in most introductory Calculus courses.

Three different conditions can occur at a stationary point:

1. local max.
2. local min
3. point of inflection (P.O.I.)

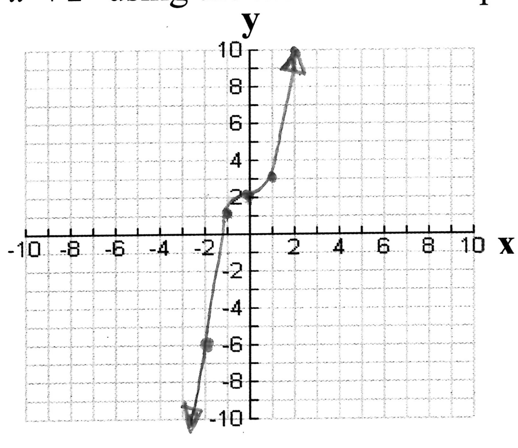




Example 1

Graph the function $f(x) = x^3 + 2$ using the table of values provided.

x	f(x)
-3	-25
-2	-6
-1	1
0	2
1	3
2	10
3	29



Calculate the IROC at and around the function when x = 0 to determine if this location is a stationary point and identify its type (if applicable).

a) x = -1

$$IROC \cong \frac{f(-0.99) - f(-1)}{0.01}$$

$$\cong \frac{1.029701 - 1}{0.01}$$

$$\cong 2.97$$

+

b) x = 0

$$IROC \cong \frac{f(0.01) - f(0)}{0.01}$$

$$\cong \frac{2.000001 - 2}{0.01}$$

$$\cong 0.0001$$

approx. zero

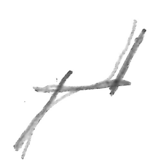
c) x = 1

$$IROC \cong \frac{f(1.01) - f(1)}{0.01}$$

$$\cong \frac{3.030301 - 3}{0.01}$$

$$\cong 3.03$$

+

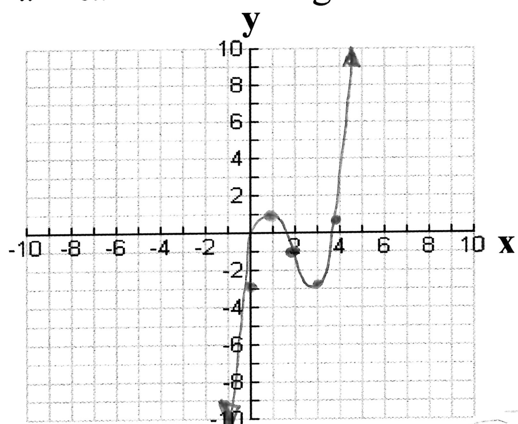


∴ The stat. point is a point of inflection.

Example 2

Graph the function $f(x)=x^3-6x^2+9x-3$ using the table of values provided.

x	f(x)
-1	-19
0	-3
1	1
2	-1
3	-3
4	1
5	17



1) Calculate the IROC at and around the function when x = 1 to determine if this location is a stationary point and identify its type (if applicable).

a) x = 0

$IROC \cong \frac{f(0.01)-f(0)}{0.01}$

$\cong \frac{-2.910599-(-3)}{0.01}$

$\cong 8.94$

$+$

b) x = 1

$IROC \cong \frac{f(1.01)-f(1)}{0.01}$

$\cong \frac{0.999701-1}{0.01}$

$\cong -0.0299$

approx. zero

c) x = 2

$IROC \cong \frac{f(2.01)-f(2)}{0.01}$

$\cong \frac{-1.02999-(-1)}{0.01}$

$\cong -2.99$

$-$

∴ The stat. point @ x = 1 is a local max

2) Calculate the IROC at and around the function when x = 3 to determine if this location is a stationary point and identify its type (if applicable).

a) x = 2

$IROC \cong -2.99$

$-$

b) x = 3

$IROC \cong \frac{f(3.01)-f(3)}{0.01}$

$\cong \frac{-2.999699-(-3)}{0.01}$

$\cong 0.0301$

approx zero

c) x = 4

$IROC \cong \frac{f(4.01)-f(4)}{0.01}$

$\cong \frac{1.090601-1}{0.01}$

$\cong 9.06$

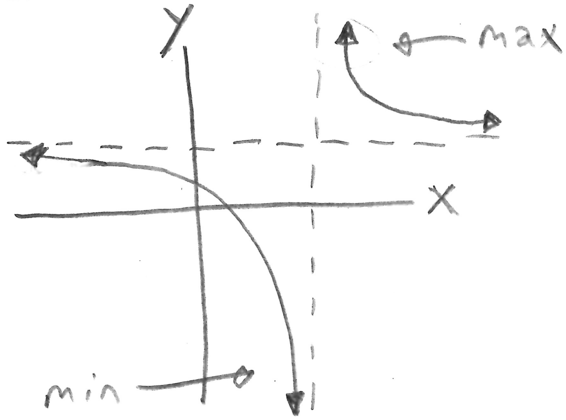
$+$

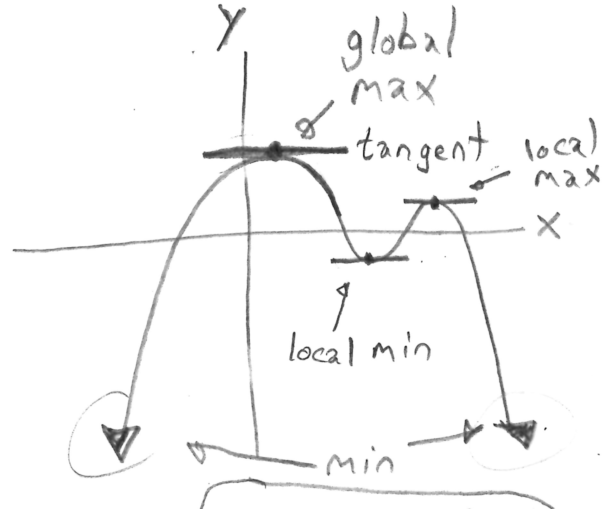
∴ The stat. point @ x = 3 is a local minimum

Note:
At a local maximum, the IROC is zero. The IROC is positive to its left and negative to its right.
At a local minimum, the IROC is zero. The IROC is negative to its left and positive to its right.
At a point of inflection, the IROC is zero. The IROC has the same sign to the left and right of the stationary point.

Optimization

Where do we find maximums and minimums?





1. Edge of Asymptotes

2. End Points
(typically $x \to \pm\infty$)

3. Stationary Point