

Solutions

Solving Trigonometric Equations - Extra Practice

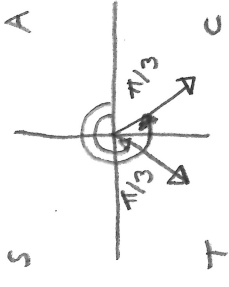
Solve the following equations; $0 \leq \theta \leq 2\pi$.

Six solutions

a) $\sin(3\theta) = -\frac{\sqrt{3}}{2}$



R.A. = $\frac{\pi}{3}$



$\frac{1}{3}(3\theta) = \frac{4\pi}{3} \Rightarrow \theta_1 = \frac{4\pi}{9}$

$\frac{1}{3}(3\theta) = \frac{10\pi}{3} \Rightarrow \theta_3 = \frac{10\pi}{9}$

$\frac{1}{3}(3\theta) = \frac{16\pi}{3} \Rightarrow \theta_5 = \frac{16\pi}{9}$

$\frac{1}{3}(3\theta) = \frac{5\pi}{3} \Rightarrow \theta_2 = \frac{5\pi}{9}$

$\frac{1}{3}(3\theta) = \frac{11\pi}{3} \Rightarrow \theta_4 = \frac{11\pi}{9}$

$\frac{1}{3}(3\theta) = \frac{17\pi}{3} \Rightarrow \theta_6 = \frac{17\pi}{9}$

b) $2\cos(2\theta) + \sin\theta + 1 = 0$

$2(1 - 2\sin^2\theta) + \sin\theta + 1 = 0$

$0 = 4\sin^2\theta - \sin\theta - 3$

Let $n = \sin\theta$

$0 = 4n^2 - n - 3$

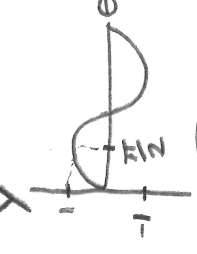
$0 = 4n^2 - 4n + 3n - 3$

$0 = 4n(n-1) + 3(n-1)$

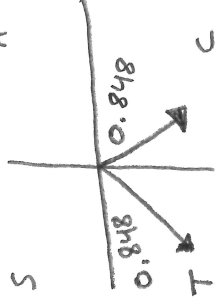
$0 = (4n+3)(n-1)$

$n = -\frac{3}{4}$ or $n = 1$

$\sin\theta = -\frac{3}{4}$



$\theta_3 = \frac{\pi}{2}$



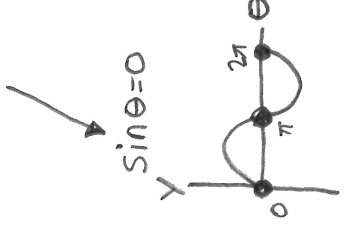
$\theta_1 = \pi + 0.848$

$\theta_2 \approx 3.990$

$\theta_4 = 2\pi - 0.848$

$\theta_5 \approx 5.435$

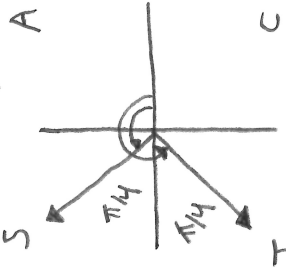
c) $\sin\theta(2\cos\theta + \sqrt{2}) = 0$



$\theta_1 = 0$
 $\theta_2 = \pi$
 $\theta_3 = 2\pi$

$2\cos\theta + \sqrt{2} = 0$
 $\frac{2\cos\theta}{2} = -\frac{\sqrt{2}}{2}$
 $\cos\theta = -\frac{\sqrt{2}}{2}$

R.A. = $\frac{\pi}{4}$

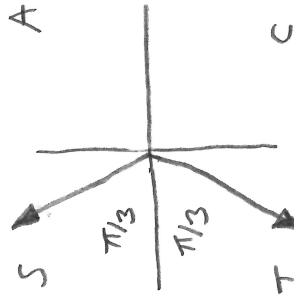


$\theta_4 = \frac{3\pi}{4}$

$\theta_5 = \frac{5\pi}{4}$

d) $\cos(2\theta) = -\frac{1}{2}$

R.A. = $\frac{\pi}{3}$



$\frac{1}{2}(2\theta) = \left(\frac{2\pi}{3}\right)\frac{1}{2}$

$\theta_1 = \frac{\pi}{3}$

$\frac{1}{2}(2\theta) = \left(\frac{4\pi}{3}\right)\frac{1}{2}$

$\theta_2 = \frac{2\pi}{3}$

$\frac{1}{2}(2\theta) = \left(\frac{8\pi}{3}\right)\frac{1}{2}$

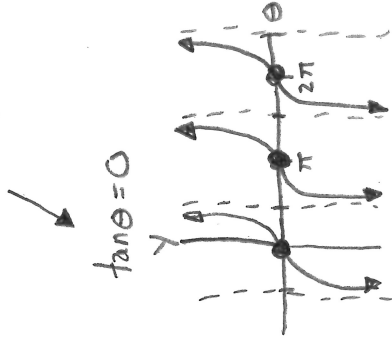
$\theta_3 = \frac{4\pi}{3}$

$\frac{1}{2}(2\theta) = \left(\frac{10\pi}{3}\right)\frac{1}{2}$

$\theta_4 = \frac{5\pi}{3}$

e) $\tan\theta \sec\theta - 3\tan\theta = 0$

$\tan\theta(\sec\theta - 3) = 0$



$\theta_1 = 0$

$\theta_2 = \pi$

$\theta_3 = 2\pi$

$\sec\theta - 3 = 0$

$\sec\theta = 3$

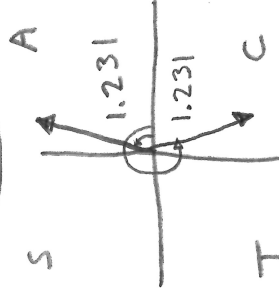
$\frac{1}{\cos\theta} = \frac{3}{1}$

$3\cos\theta = \frac{1}{3}$

$\cos\theta = \frac{1}{3}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$

$\theta_4 \approx 1.231$



$\theta_5 = 2\pi - 1.231$

$\theta_5 \approx 5.052$

f) $\cos\theta + 1 = \frac{2}{3}\sin^2\theta$

$3[\cos\theta + 1] = \left[\frac{2}{3}(1 - \cos^2\theta)\right]3$

$3\cos\theta + 3 = 2(1 - \cos^2\theta)$

$3\cos\theta + 3 = 2 - 2\cos^2\theta$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

Let $n = \cos\theta$

$2n^2 + 3n + 1 = 0$

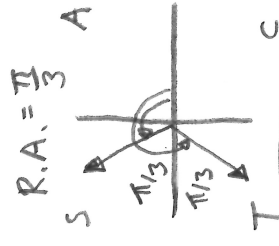
$2n^2 + 2n + 1n + 1 = 0$

$2n(n+1) + 1(n+1) = 0$

$(2n+1)(n+1) = 0$

$n = -\frac{1}{2}$ or $n = -1$

$\cos\theta = -\frac{1}{2}$



$\theta_1 = \frac{2\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

Answers: a) $\frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

b) 5.435, 3.990, $\frac{\pi}{2}$

c) $0, \pi, 2\pi, \frac{3\pi}{4}, \frac{5\pi}{4}$

d) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

e) $0, \pi, 2\pi, 1.231, 5.052$

f) $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$