Hmwk: pg 285 #1, 2-7(ace), 9

Solving Rational Equations

There are many strategies used to solve rational equations:

1. Cross multiplication
2. Multiplying Through
3. Common Denominator
4. Graphing

Cross Multiplication

If an algebraic equation can easily be reduced to a single fraction on both sides of an equal sign, then cross multiplication can by an optimal technique to use.

$\frac{—}{} = \frac{—}{}$

Solve the following.

a) $\frac{4}{x} = \frac{3}{15}$

$\frac{3x}{3} = \frac{60}{3}$

$\boxed{x = 20}$

b) $\frac{x-1}{x+3} = \frac{3}{6}$

$6(x-1) = 3(x+3)$

$6x - 6 = 3x + 9$

$6x - 3x = 9 + 6$

$\frac{3x}{3} = \frac{15}{3}$

$\boxed{x = 5}$

c) $\frac{x+2}{5x} - \frac{(x+3)5}{(x)5} = -\frac{9}{8}$

$\frac{(x+2) - 5(x+3)}{5x} = \frac{-9}{8}$

$\frac{-4x-13}{5x} = -\frac{9}{8}$

$-32x - 104 = -45x$

$-32x + 45x = 104$

$\frac{13x}{13} = \frac{104}{13}$

$\boxed{x = 8}$

Multiplying Through

If single fractions cannot easily be made on both sides of a fraction, consider multiplying through by a common denominator.

Solve the following.

a) $\overset{12x}{\frac{x}{3}} + \overset{12x}{\frac{x}{4}} = \overset{12x}{\frac{7}{3x}}$

$4x^2 + 3x^2 = 28$

$\frac{7x^2}{7} = \frac{28}{7}$

$\sqrt{x^2} = \pm\sqrt{4}$

$\boxed{x = \pm 2}$

b) $\overset{x(x+1)}{\frac{3}{x}} + \overset{x(x+1)}{\frac{4}{x+1}} = \overset{x(x+1)}{\frac{2}{1}}$

$3(x+1) + 4x = 2x(x+1)$

$7x + 3 = 2x^2 + 2x$

$0 = 2x^2 + 5x - 3$

$-6, 1 \begin{cases} P(-6) \\ S(5) \end{cases}$

$0 = 2x^2 - 6x + x - 3$

$0 = 2x(x-3) + 1(x-3)$

$0 = (2x+1)(x-3)$

$\boxed{x = -\frac{1}{2}}$ or $\boxed{x = 3}$

c) $\overset{x(1-x)}{\frac{1}{x}} = \overset{x(1-x)}{\frac{2}{x}} + \overset{x(1-x)}{\frac{1}{1}} + \overset{x(1-x)}{\frac{1}{1-x}}$

$1 - x = 2(1-x) + x(1-x) + x$

$1 - x = 2 - 2x + \cancel{x} - x^2 + \cancel{x}$

$x^2 - x - 1 = 0$

$b^2 - 4ac = (-1)^2 - 4(1)(-1) = 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{1 \pm \sqrt{5}}{2}$

$\boxed{x \cong 1.62}$ or $\boxed{x \cong -0.62}$

Common Denominator

Sometimes, the denominators of all components in a rational equation are all very similar. In these cases, it may be ideal to use a common denominator.

Solve the following.

a) $\frac{2(1)}{2(x)}+\frac{3}{2x}=\frac{9}{2x}$

$\frac{5}{2x}=\frac{9}{2x}$

$18x=10x$

$18x-10x=0$

$\frac{8x}{8}=\frac{0}{8}$

$\boxed{\cancel{x=0}}$

↑ inadmissable

b) $\frac{5}{x+2}-\frac{9}{x^2+5x+6}=\frac{1}{x+3}$

$\frac{(x+3)(5)}{(x+3)(x+2)}-\frac{9}{(x+2)(x+3)}=\frac{1(x+2)}{(x+3)(x+2)}$

$\frac{5x+15-9}{(x+3)(x+2)}=\frac{x+2}{(x+3)(x+2)}$

$5x+6=x+2$

$5x-x=2-6$

$\frac{4x}{4}=\frac{-4}{4}$

$\boxed{x=-1}$

c) $\frac{x-2}{x^2+2x-8}=\frac{x^2+3x}{x^2+4x}$

$\frac{\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}=\frac{\cancel{x}(x+3)}{\cancel{x}(x+4)}$

$\frac{1}{x+4}=\frac{x+3}{x+4}$, $x\neq 0,2$

$1=x+3$

$-2=x$

$\boxed{x=-2}$

Graphing

The graphing method can be an ideal choice for solving a rational equation when a graphing calculator or software is readily available.

Solve the following equation by graphing.

$3(x+2)=1(x-6)$

$3x+6=x-6$

$\frac{2x}{2}=\frac{-12}{2}$

$\boxed{x=-6}$

$\frac{1}{x+2}=\frac{3}{x-6}$

$0=\frac{(3)(x+2)}{(x-6)(x+2)}-\frac{(1)(x-6)}{(x+2)(x-6)}$

$0=\frac{2x+12}{(x-6)(x+2)}$

$0=\frac{2(x+6)}{(x-6)(x+2)}$

Graph → $y=\frac{2(x+6)}{(x-6)(x+2)}$

V.A. @ $x=6,-2$

H.A. @ $y=0$

x-int (y=0)

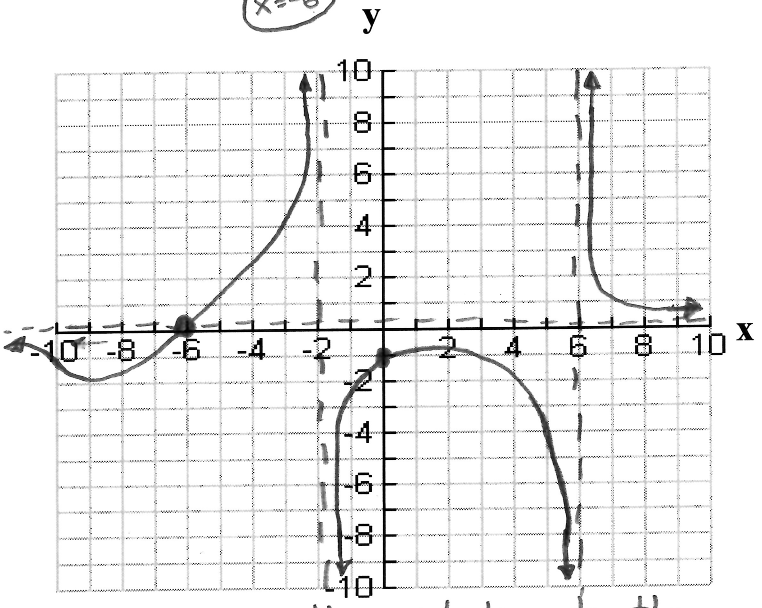
$2(x+6)=0$

x-int $=-6$

y-int (x=0)

$y=\frac{2(6)}{(-6)(2)}$

y-int $=-1$



Since the x-int $=-6$, the solution to the original equation is $\boxed{x=-6}$