Homework: pg 435 #1abc, 2ac, 3, 4-8ace, 9ac, 10, (11), 12, 14, 15a, (18)

# Solving Quadratic Trigonometric Equations

Prior Knowledge:

There are several methods to solve a quadratic equation:

1. Factoring
2. Quadratic Formula
3. Graphing

A quadratic equation may have two solutions, one solution or no solution.

$0 = x^2 + 2x - 8$
$y = x^2 + 2x - 8$
$y = (x^2 + 2x + 1 - 1) - 8$
$y = (x^2 + 2x + 1) - 8 - 1$
$y = (x+1)^2 - 9$
vertex → $(-1, -9)$
step pattern: 1, 3, 5, 7

**Activity 1**

Solve

~~Consider~~ the equation $x^2 + 2x - 8 = 0$ using multiple techniques.

Factoring

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ or $x = 2$

Use Zero Principle!

* $A \cdot B = 0$
Solns → $A = 0$, $B = 0$

Quadratic Formula

$x^2 + 2x - 8 = 0$

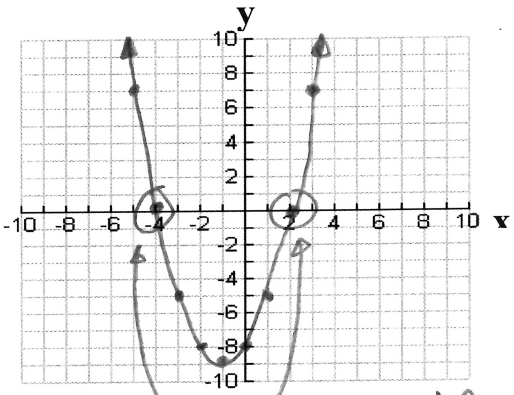
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-8)}}{2(1)}$

$x = \dfrac{-2 \pm 6}{2}$

$x = 2$ or $x = -4$

Graphing




x-ints: $-4$ and $2$

∴ The solns are $-4$ and $2$

**Example 1**

Solve the equation $2\cos^2\theta - 1 = 0$; $0 \le \theta \le 2\pi$.

$2\cos^2\theta - 1 = 0$

$\dfrac{2\cos^2\theta}{2} = \dfrac{1}{2}$

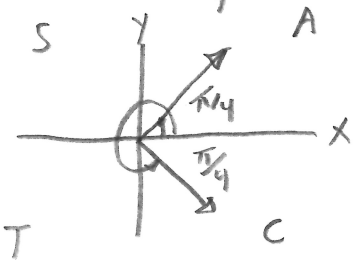
$\sqrt{\cos^2\theta} = \pm\sqrt{\dfrac{1}{2}}$

$\cos\theta = \pm\dfrac{1}{\sqrt{2}}\dfrac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \pm\dfrac{\sqrt{2}}{2}$

$\cos\theta = \dfrac{\sqrt{2}}{2}$

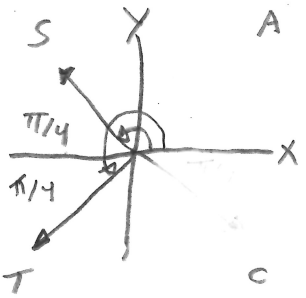
R.A. $= \dfrac{\pi}{4}$




$\theta_1 = \dfrac{\pi}{4}$ $\theta_2 = \dfrac{7\pi}{4}$

$\cos\theta = -\dfrac{\sqrt{2}}{2}$

R.A. $= \dfrac{\pi}{4}$




$\theta_3 = \dfrac{3\pi}{4}$ $\theta_4 = \dfrac{5\pi}{4}$

**Example 2**

Solve the following trigonometric equations; $0 \leq \theta \leq 2\pi$.

a) $3\cos^2\theta - 11\cos\theta = -6$

$3\cos^2\theta - 11\cos\theta + 6 = 0$

Let $n = \cos\theta$

$3n^2 - 11n + 6 = 0$

$P(18)\ S(-11)\ \}\ -9, -2$

$3n^2 - 9n - 2n + 6 = 0$

$3n(n-3) - 2(n-3) = 0$

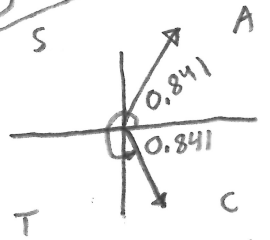
$(3n-2)(n-3) = 0$

$n = \frac{2}{3}$

$\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta_1 \approx 0.841$

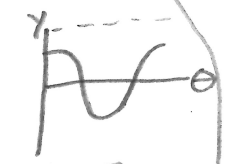



$\theta_2 = 2\pi - 0.841$

$\theta_2 \approx 5.442$

$n = 3$

$\cos\theta = 3$




No Soln

b) $-2\cos^2\theta - 3\sin\theta + 3 = 0$

$-2(1-\sin^2\theta) - 3\sin\theta + 3 = 0$

$-2 + 2\sin^2\theta - 3\sin\theta + 3 = 0$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

Let $n = \sin\theta$

$2n^2 - 3n + 1 = 0$

$P(2)\ S(-3)\ \}\ -2, -1$

$2n^2 - 2n - 1n + 1 = 0$

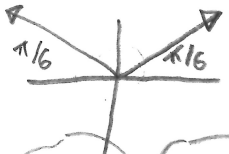
$2n(n-1) - 1(n-1) = 0$

$(2n-1)(n-1) = 0$

$n = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

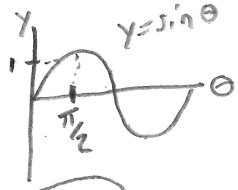
R.A. $= \pi/6$




$\theta_1 = \frac{\pi}{6}$ $\quad \theta_2 = \frac{5\pi}{6}$

$n = 1$

$\sin\theta = 1$




$\theta_3 = \frac{\pi}{2}$

**Example 3**

Solve the following trigonometric equations; $0 \leq \theta \leq 2\pi$.

Hint: Factoring may help.

Use Zero Principle!

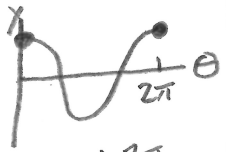
a) $\sin\theta\,(\cos\theta - 1) = 0$

$\sin\theta = 0$


$\theta_1 = 0$, $\theta_2 = \pi$, $\theta_3 = 2\pi$

$\cos\theta - 1 = 0$

$\cos\theta = 1$




0 and $2\pi$ are already listed as solutions

b) $\sec\theta\csc\theta - 2\csc\theta = 0$

$\csc\theta(\sec\theta - 2) = 0$

$\csc\theta = 0$

$\frac{1}{\sin\theta} = \frac{0}{1}$

can't be zero...

∴ No Soln

$\sec\theta - 2 = 0$

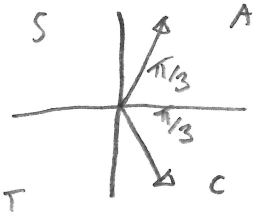
$\sec\theta = 2$

$\frac{1}{\cos\theta} = \frac{2}{1}$

$\frac{2\cos\theta}{2} = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

R.A. $= \frac{\pi}{3}$




$\theta_1 = \frac{\pi}{3}$ $\quad \theta_2 = \frac{5\pi}{3}$

**Example 4**

How would you start a solution to the following equations below?

a) $5\cos 2\theta - \cos\theta + 3 = 0$

$5(2\cos^2\theta - 1) - \cos\theta + 3 = 0$

$10\cos^2\theta - 5 - \cos\theta + 3 = 0$

$10\cos^2\theta - \cos\theta - 2 = 0$

Let $n = \cos\theta$

b) $-2\cos 2\theta = 2\sin\theta$

$-2(1 - 2\sin^2\theta) = 2\sin\theta$

$-2 + 4\sin^2\theta = 2\sin\theta$

$\frac{4\sin^2\theta - 2\sin\theta - 2}{2} = \frac{0}{2}$

$2\sin^2\theta - \sin\theta - 1 = 0$

Let $n = \sin\theta$

c) $\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta = \cos\theta$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\quad 2\sin\theta - 1$