

## Solving Polynomial Inequalities

Polynomial inequalities can be solved using two methods:

1. Factors/Sign chart (Number Line Strategy)
2. Graphing

### Factors Chart/Number Line Strategy

1. Move all terms to one side of the inequality.
2. Factor the polynomial.
3. List the ordered solutions of  $x$  along the top of the chart.
4. List the factors down the left.
5. Determine the product of the factors in each column.

### Example 1

Solve  $-2x^3 + 4x^2 + 38x - 40 \leq 0$  using a factors chart.

$$-2(x^3 - 2x^2 - 19x + 20) \leq 0$$

$F(x)$   
 $F(1) = 0$   
 $\therefore x-1$  is a factor

1	1	-2	-19	20
		1	-1	-20
1	-1	-20	0	

$$-2(x-1)(x^2 - x - 20) \leq 0$$

$$-2(x-1)(x-5)(x+4) \leq 0$$

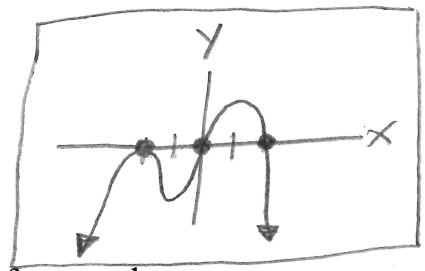
Solns if this expression were equal to zero are 1, 5, -4

For  $\leq 0$ ,  
choose the  $\ominus$

	$x = -4$	$x = 1$	$x = 5$	
$-2$	-	-	-	-
$x-1$	-	-	+	+
$x-5$	-	-	-	+
$x+4$	-	+	+	+
	+	$\ominus$	+	$\ominus$

$-4 \leq x \leq 1$  or  $x \geq 5$

$$y = -2x(x-2)(x+2)^2$$



### Example 2

Solve  $-4x^4 - 4x^3 + 6x^2 < -2x^4 - 2x^2 - 16x$  using a factors chart.

$$\begin{aligned} -2x^4 - 4x^3 + 8x^2 + 16x < 0 \\ -2x(x^3 + 2x^2 - 4x - 8) < 0 \\ -2x[x^2(x+2) - 4(x+2)] < 0 \\ -2x(x^2 - 4)(x+2) < 0 \\ -2x(x-2)(x+2)(x+2) < 0 \\ -2x(x-2)(x+2)^2 < 0 \end{aligned}$$

look for  $\ominus$

	$x < -2$	$x = 0$	$x = 2$	
$-2x$	+	+	-	-
$x-2$	-	-	-	+
$(x+2)^2$	+	+	+	+
	$\ominus$	$\ominus$	+	$\ominus$

$x < -2, -2 < x < 0, \text{ or } x > 2$

### Graphing Method

1. Move all terms to one side of the inequality.
2. Factor the polynomial.
3. Use your knowledge of polynomial functions to sketch a graph of the factored expression.
4. Examine the graph to see where the function is positive or negative.

### Examine 3

Solve  $5x^3 - 21x < x^4 + x^2 - 18$  using the graphing method.

$$\begin{aligned} -x^4 + 5x^3 - x^2 - 21x + 18 < 0 \\ -(x^4 - 5x^3 + x^2 + 21x - 18) < 0 \\ \hline \phantom{-} x^4 - 5x^3 + x^2 + 21x - 18 > 0 \end{aligned}$$

$$\underbrace{x^4 - 5x^3 + x^2 + 21x - 18}_{f(x)} > 0 \quad \leftarrow \text{positive}$$

$$\begin{array}{r|rrrrr} 1 & 1 & -5 & 1 & 21 & -18 \\ & & & & & \\ \hline & 1 & -4 & -3 & 18 & \\ 1 & -4 & -3 & 18 & 0 & \end{array}$$

$$(x-1)(x^3 - 4x^2 - 3x + 18) > 0$$

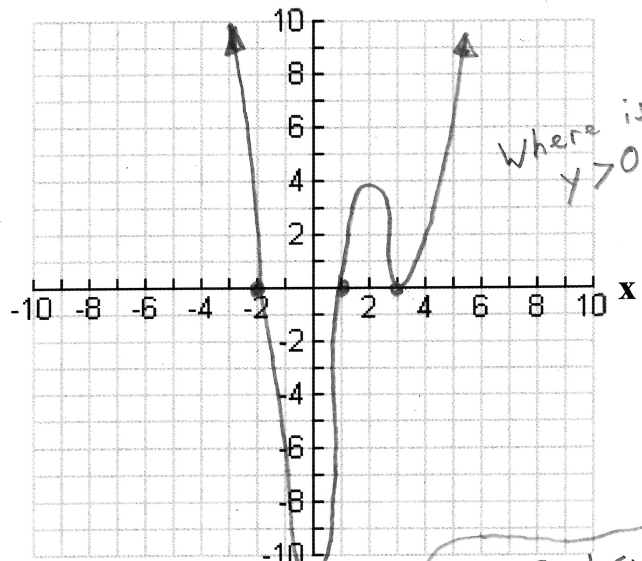
$$\begin{aligned} g(x) \\ g(1) = 12 \\ g(3) = 0 \\ \therefore x-3 \text{ is a factor} \end{aligned}$$

$$\begin{array}{r|rrrrr} 3 & 1 & -4 & -3 & 18 \\ & & & & & \\ \hline & 1 & -1 & -6 & 0 & \end{array}$$

$$(x-1)(x-3)(x^2 - x - 6) > 0$$

$$\begin{aligned} (x-1)(x-3)(x-3)(x+2) > 0 \\ (x-1)(x-3)^2(x+2) > 0 \end{aligned}$$

graph  $y = (x-1)(x-3)^2(x+2)$   
 x-ints: 1, 3, -2  
 y-int: -18  
 end as  $x \rightarrow -\infty, y \rightarrow \infty$   
 as  $x \rightarrow \infty, y \rightarrow \infty$



$x < -2, 1 < x < 3, \text{ or } x > 3$