Hmwk: pg 204 #1-2(ace), 5, 6,7ace,9ac, 13, 15, (16), (18)

Solving Polynomial Equations Algebraically

Recall:

We have seen in the past that a polynomial equation of degree two can be solved using the quadratic formula or by factoring.

Example 1

Solve the following equation using two methods.

$6x^2 + 15x = 9$

Method 1 (Quadratic Formula)

$6x^2+15x-9=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-15\pm\sqrt{(15)^2-4(6)(-9)}}{2(6)}$

$=\frac{-15\pm 21}{12}$

$=\frac{1}{2}$ or -3

* Method 2 (Factoring)

$6x^2+15x-9=0$

$3(2x^2+5x-3)=0$ $\quad P(-6), S(5) \} 6,-1$

$3(2x^2+6x-1x-3)=0$

$3[2x(x+3)-1(x+3)]=0$

$3(2x-1)(x+3)=0$

$2x-1=0$ $\quad x=-3$

$\frac{2x}{2}=\frac{1}{2}$

$x=\frac{1}{2}$

Notice that both methods require that a Zero be put on one side of the equation to solve it.

The second method (factoring) makes use of the "Zero Principle" which states that if any factor in the factored expression can be made to be zero by assigning some value to x then the entire factored expression becomes zero and this value of x is, therefore, a solution.

The Zero Principle
If a polynomial equation can be written as a product as follows: $k(x - a_1)(x - a_2)(x - a_3)....(x - a_n) = 0$ where k, $a_1, a_2, a_3,..., a_n$ are all constants, then the solutions (or roots) to the equation are $a_1, a_2, a_3,..., a_n$.

The Zero Principle can also be used to solve one variable polynomial equations of degree larger than 2.

Example 2

Solve each equation using factoring techniques.

a) $x^3 - 4x^2 - 5x = 0$

$x(x^2-4x-5)=0$

$(x-0)$

$x(x-5)(x+1)=0$

$x = 0, 5, -1$

b) $4x^3 - 12x^2 - 1x + 3 = 0$

$\frac{-12}{4} = -3$ $\quad \frac{3}{-1} = -3$

Use grouping!

$4x^2(x-3)-1(x-3)=0$

$(4x^2-1)(x-3)=0$

$(2x-1)(2x+1)(x-3)=0$

$x = \pm\frac{1}{2}, 3$

c) $x^3 - 5x = 2x^2 - 6$

$\underbrace{x^3-2x^2-5x+6}_{f(x)}=0$

$f(1)=0$

$\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$(x-1)(x^2-x-6)=0$

$(x-1)(x-3)(x+2)=0$

$x = 1, 3, -2$

d) $\overbrace{x^3 - 4x^2 + 8x - 8}^{f(x)} = 0$

$f(1)=-3$

$f(-1)=-21$

$f(2)=0$ ✓

$\therefore x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$(x-2)\underbrace{(x^2-2x+4)}=0$

$x=2$

b^2-4ac

$=(-2)^2-4(1)(4)$

$=-12$

$\therefore$ Not factorable.

e) $x^4 + 4 = 5x^2$

$x^4-5x^2+4=0$

Let $n=x^2$

$n^2-5n+4=0$

$(n-1)(n-4)=0$

$n=1$ or $n=4$

$\sqrt{x^2}=\pm\sqrt{1}$ $\quad \sqrt{x^2}=\pm\sqrt{4}$

$x=\pm 1$ $\quad x=\pm 2$

f) $\underbrace{x^4 + 2x^3 + 14x^2 + 13x + 36}_{f(x)} = 0$

$f(1)=66$

$f(-1)=36$

$f(2)=150$

$f(-2)=66$

$f(3)=336$

$f(-3)=150$

$f(4)=696$

$f(-4)=336$

⋮

Give Up

This has no roots/sol^{ns}.

How do we know?

Use graphing technology!