Homework: pg 491# 1ace, 2ace, 3, 4-5(ace), 6, 7-8(ace), 9, 12, 15, 16

Solving Logarithmic Equations

Recall: Logarithmic Laws

$\log_a m\,n = \log_a m + \log_a n$

$\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_a m^n = n \log_a m$

Restrictions on Logarithmic Functions

The argument and base of a logarithmic function must always be positive.

Determine the possible value(s) for the following logarithmic expressions:

a) $\log_5(3x - 8)$

$3x - 8 > 0$

$\frac{3x}{3} > \frac{8}{3}$

$x > \frac{8}{3}$

b) $\log_{5-x}(x - 2)$

$5 - x > 0$ $\quad$ $x - 2 > 0$

$-x > -5$ $\quad$ $x > 2$

$-1 \quad -1$

$x < 5$

$2 < x < 5$

Practice

Solve the following logarithmic equations.

a) $\log_3 x = \log_3 5$

"unlog" remove $\log_3$ from both sides

$x = 5$

b) $\log_x 16 = -2$

$x^{-2} = 16$

$\frac{1}{x^2} = \frac{16}{1}$

$\frac{16x^2}{16} = \frac{1}{16}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{16}}$

$x = \frac{1}{4}$ or $x = -\frac{1}{4}$ ← inadmissable

c) $\log_3(2x-5) = \log_3 9$

$2x-5=9$

$2x=9+5$

$\frac{2x}{2}=\frac{14}{2}$

$x=7$

d) $\log_3 x - \log_3 5 = \log_3 4$

$\log_3\left(\frac{x}{5}\right)=\log_3 4$

$\frac{x}{5}=\frac{4}{1}$

$x=20$

e) $\log x^3 - \log x = 2$

$\log\left(\frac{x^3}{x}\right)=2$ or $3\log x-\log x=2$

$\log(x^2)=2$ $\quad\frac{2\log x}{2}=\frac{2}{2}$

$\log_{10}(x^2)=2$ $\quad\log x=1$

$x^2=10^2$ $\quad x=10^1$

$\sqrt{x^2}=\pm\sqrt{100}$

$x=\pm 10$

inadmissible

$x\neq -10$

f) $\log_2(x-4)+\log_2(x+2)=4$

$\log_2[(x-4)(x+2)]=4$

$\log_2[x^2-2x-8]=4$

$x^2-2x-8=2^4$

$x^2-2x-8=16$

$x^2-2x-24=0$

$(x-6)(x+4)=0$

inadmissable

$x=6$ or $x\neq -4$

g) $\log_5 x + \log_7 x = 6$

$\frac{\log 7}{\log 7}\frac{\log x}{\log 5}+\frac{\log x}{\log 7}\frac{\log 5}{\log 5}=6$

$\frac{\log 7\log x+\log x\log 5}{\log 7\log 5}=\frac{6}{1}$

$\log x(\log 7+\log 5)=6\log 7\log 5$

$(\log 7+\log 5)\qquad\log 7+\log 5$

$\log_{10} x=\frac{6\log 7\log 5}{\log 7+\log 5}$

$x=10^{\frac{6\log 7\log 5}{\log 7+\log 5}}$

$x\cong 197.40$

h) $\log_x 10 = \log_{10000} x$

$\frac{\log 10}{\log x}=\frac{\log x}{\log 10000}$

$\frac{1}{\log x}=\frac{\log x}{4}$

$\sqrt{(\log x)^2}=\pm\sqrt{4}$

$\log x=\pm 2$

$x=10^2$ $\quad x=10^{-2}$

$x=100$ $\quad x=\frac{1}{100}$