Hmwk: pg 426 #1abc, 2abc, 3, 4, 6ace, 8ab, 10ad, 12, (13), (17)

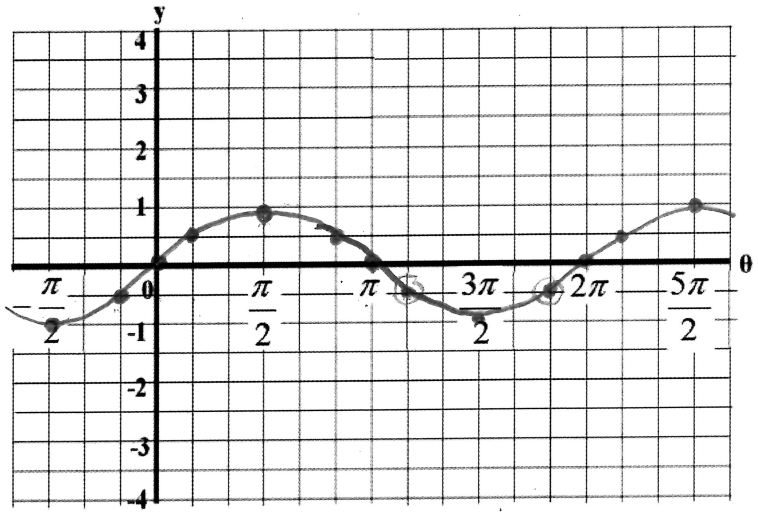
Solving Linear Trigonometric Equations

Recall: SYR CXR TYX

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Example 1

Create a sketch of the function y = sinθ.



Use

~~Using~~ the above graph to "estimate" a solution for the following: $0 \le \theta \le 2\pi$.

a) $\sin\theta = -\frac{1}{2}$

$\sin\theta = y$ $y = -\frac{1}{2}$

$\theta_1 = \frac{7\pi}{6}$ or $\theta_2 = \frac{11\pi}{6}$

b) $5\sin\theta - 1 = 4$

$5\sin\theta = 4+1$

$\frac{5\sin\theta}{5} = \frac{5}{5}$

$\sin\theta = 1$

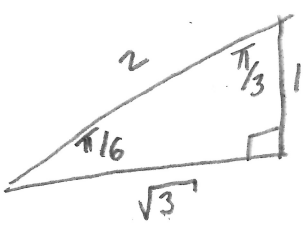
$\theta = \frac{\pi}{2}$

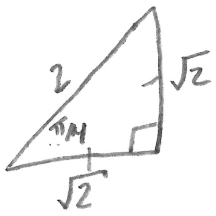
c) $3\sin\theta - 5 = 1$

$\frac{3\sin\theta}{3} = \frac{6}{3}$

$\sin\theta = 2$

No Sol^{n}





Example 2

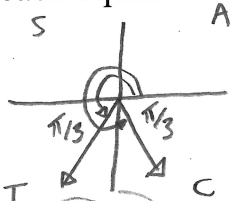
Use your knowledge of special triangles and the CAST rule to determine the exact solution for each equation below: $0 \le \theta \le 2\pi$.

a) $2\sin\theta + \sqrt{3} = 0$

$\frac{2\sin\theta}{2} = \frac{-\sqrt{3}}{2}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$RA = \frac{\pi}{3}$



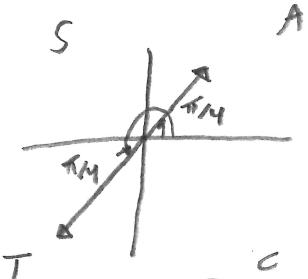
$\theta_1 = \frac{4\pi}{3}$ or $\theta_2 = \frac{5\pi}{3}$

b) $\frac{3(\tan\theta - 3)}{3} = \frac{-6}{3}$

$\tan\theta - 3 = -2$

$\tan\theta = 1$

$R.A. = \frac{\pi}{4}$



$\theta_1 = \frac{\pi}{4}$ or $\theta_2 = \frac{5\pi}{4}$

Example 3

Solve the following trigonometric function; $0 \le \theta \le 2\pi$.

a) $6\sin\theta\cos\theta = \sin 2\theta + 1$

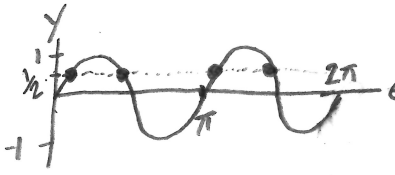
$3(2\sin\theta\cos\theta) = \sin 2\theta + 1$

$3\sin 2\theta = \sin 2\theta + 1$

$3\sin 2\theta - 1\sin 2\theta = 1$

$\frac{2\sin 2\theta}{2} = \frac{1}{2}$

$\sin 2\theta = \frac{1}{2}$



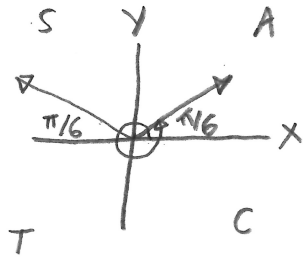
$\frac{1}{2}(2\theta) = \left(\frac{\pi}{6}\right)\frac{1}{2}$

$\theta_1 = \frac{\pi}{12}$

$\frac{1}{2}(2\theta) = \left(\frac{5\pi}{6}\right)\frac{1}{2}$

$\theta_2 = \frac{5\pi}{12}$

$R.A. = \frac{\pi}{6}$



$\frac{1}{2}(2\theta) = \left(\frac{13\pi}{6}\right)\frac{1}{2}$

$\theta_3 = \frac{13\pi}{12}$

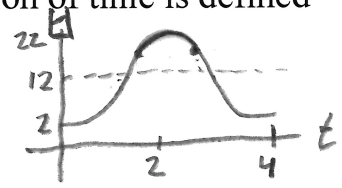
$\frac{1}{2}(2\theta) = \left(\frac{17\pi}{6}\right)\frac{1}{2}$

$\theta_4 = \frac{17\pi}{12}$

Example 4

Avalon boards a Ferris wheel. Her height, H, as a function of time is defined by:

$$H(t) = -10\cos\left(\frac{\pi t}{2}\right) + 12$$



where 't' is the time in minutes.

If the wheel only makes one turn, how long was Avalon above a height of 16 metres?

$T = \frac{2\pi}{|k|}$

$= \frac{2\pi}{\pi/2}$

$= \frac{2\pi}{1} \cdot \frac{2}{\pi}$

$= 4$

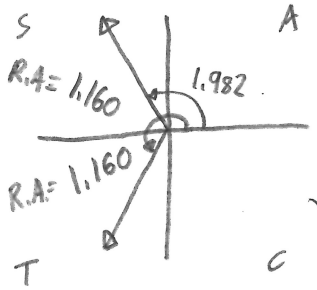
Set $H = 16$

$-10\cos\left(\frac{\pi t}{2}\right) + 12 = 16$

$\frac{-10\cos\left(\frac{\pi t}{2}\right)}{-10} = \frac{4}{-10}$

$\cos\left(\frac{\pi t}{2}\right) = -0.4$

$\frac{\pi t}{2} = \cos^{-1}(-0.4)$



$\frac{\pi t}{2} = \frac{1.982}{1}$

$\frac{\pi t}{\pi} = \frac{2(1.982)}{\pi}$

$t_1 = 1.26$

$\frac{\pi t}{2} = \frac{4.302}{1}$

$\frac{\pi t}{\pi} = \frac{2(4.302)}{\pi}$

$t_2 = 2.74$

$\Delta t = t_2 - t_1$

$= 2.74 - 1.26$

$= 1.48$ minutes

$=$ 1 minute 29 seconds