Homework: pg 485 #1-3(ace), 4a, 5ace, 6ab, 7, 8ace, *12, 14, 16

Solving Exponential Equations

Example 1

Solve the following exponential equation using multiple techniques.

$$10 = 80(2)^{\frac{3}{x}}$$

Method 1 (Using Logarithms)

$\frac{10}{80} = \frac{80(2)^{\frac{3}{x}}}{80}$

$\frac{1}{8} = 2^{3/x}$

$\log \frac{1}{8} = \log 2^{3/x}$

$\frac{\log \frac{1}{8}}{\log 2} = \frac{\frac{3}{x}\log 2}{\log 2}$

$\frac{\log \frac{1}{8}}{\log 2} = \frac{3}{x}$

$\frac{x \log \frac{1}{8}}{\log \frac{1}{8}} = \frac{3 \log 2}{\log \frac{1}{8}}$

$x = -1$

Method 2 (Using Common Base)

$\frac{10}{80} = \frac{80(2)^{3/x}}{80}$

$\frac{1}{8} = 2^{3/x}$

$2^{-3} = 2^{3/x}$

$\frac{-3}{1} = \frac{3}{x}$

$\frac{-3x}{-3} = \frac{3}{-3}$

$x = -1$

Example 2

Solve the following using common bases.

a) $4^{2x-3} = 8^{x+5}$

$(2^2)^{(2x-3)} = (2^3)^{(x+5)}$

$2^{4x-6} = 2^{3x+15}$

$4x - 6 = 3x + 15$

$4x - 3x = 15 + 6$

$x = 21$

b) $27^{x-2} = \sqrt[3]{81}$

$27^{x-2} = 81^{\frac{1}{3}}$

$(3^3)^{(x-2)} = (3^4)^{\frac{1}{3}}$

$3^{3x-6} = 3^{4/3}$

$\log_3 3^{3x-6} = \log_3 3^{4/3}$

$\frac{3x-6}{1} = \frac{4}{3}$

$9x - 18 = 4$

$9x = 4 + 18$

$\frac{9x}{9} = \frac{22}{9}$

$x = \frac{22}{9}$

Example 3

Solve the following exponential equations.

a) $3^{x+2} - 3^x = 18$

$3^x 3^2 = 3^{x+2}$

$3^x 3^2 - 3^x = 18$

$3^x(3^2 - 1) = 18$

$\frac{3^x(8)}{8} = \frac{18}{8}$

$3^x = \frac{9}{4}$

$x = \log_3 9/4$

$x = \frac{\log 9/4}{\log 3}$

$x \cong 0.74$

b) $4^x + 2^{2x-5} = 12$

$(2^2)^x + 2^{2x-5} = 12$

$2^{2x} + 2^{2x}2^{-5} = 12$

$2^{2x}(1 + 2^{-5}) = 12$

$2^{2x}\left(\frac{32}{32} + \frac{1}{32}\right) = 12$

$\left(\frac{32}{33}\right)2^{2x}\left(\frac{33}{32}\right) = \frac{12}{1}\left(\frac{32}{33}\right)$

$2^{2x} = \frac{384}{33}$

$\frac{2x}{2} = \log_2(384/33) \div 2$

$x = \frac{\log(384/33)}{\log 2} \div 2$

$x \cong 1.77$

Example 4

Solve the following exponential equations.

a) $5^{x-2} = 3^{x+2}$

$\log 5^{x-2} = \log 3^{x+2}$

$(x-2)\log 5 = (x+2)\log 3$

$x\log 5 - 2\log 5 = x\log 3 + 2\log 3$

$x\log 5 - x\log 3 = 2\log 3 + 2\log 5$

$\frac{x(\log 5 - \log 3)}{\log 5 - \log 3} = \frac{(2\log 3 + 2\log 5)}{(\log 5 - \log 3)}$

$x \cong 10.60$

b) $3^x 5^{x-3} = 24$

$\log 3^x 5^{x-3} = \log 24$

$\log 3^x + \log 5^{x-3} = \log 24$

$x\log 3 + (x-3)\log 5 = \log 24$

$x\log 3 + x\log 5 - 3\log 5 = \log 24$

$x\log 3 + x\log 5 = \log 24 + 3\log 5$

$\frac{x(\log 3 + \log 5)}{(\log 3 + \log 5)} = \frac{(\log 24 + 3\log 5)}{(\log 3 + \log 5)}$

$x \cong 2.96$

Example 5

Solve the following exponential equation.

$2(2^{2x}) + 7(2^x) - 4 = 0$

$2(2^x)^2 + 7(2^x) - 4 = 0$

Let $n = 2^x$

$2n^2 + 7n - 4 = 0$ $\quad P(-8), S(7) \}\ 8, -1$

$2n^2 + 8n - 1n - 4 = 0$

$2n(n+4) - 1(n+4) = 0$

$(2n-1)(n+4) = 0$

$n = \frac{1}{2}$ or $n = -4$

$n = \frac{1}{2}$

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

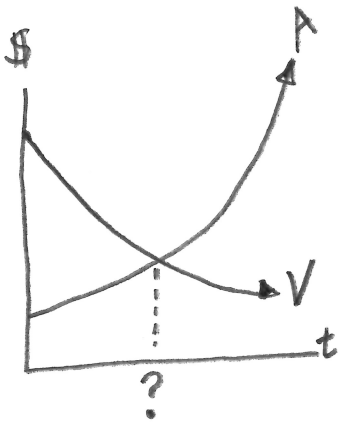
$n = -4$

$2^x = -4$

No Soln

Example 6

Marylou puts $5000 into a bank account that earns 5% compounded annually. She wishes to purchase a car that is currently worth $25,000. The car depreciates at a rate of 12% each year. When will she be able to purchase the car?



$A = P(1+i)^n$

$A = 5000(1.05)^t$

Investment

$V = a(1-r)^t$

$V = 25000(0.88)^t$

Value of the Car

set $A = V$

$\frac{5000(1.05)^t}{5000} = \frac{25000(0.88)^t}{5000}$

$1.05^t = 5(0.88)^t$

$\log 1.05^t = \log[5(0.88)^t]$

$\log 1.05^t = \log 5 + \log 0.88^t$

$t \log 1.05 = \log 5 + t \log 0.88$

$t \log 1.05 - t \log 0.88 = \log 5$

$\frac{t(\log 1.05 - \log 0.88)}{\log 1.05 - \log 0.88} = \frac{\log 5}{(\log 1.05 - \log 0.88)}$

$t \approx 9.11$ years