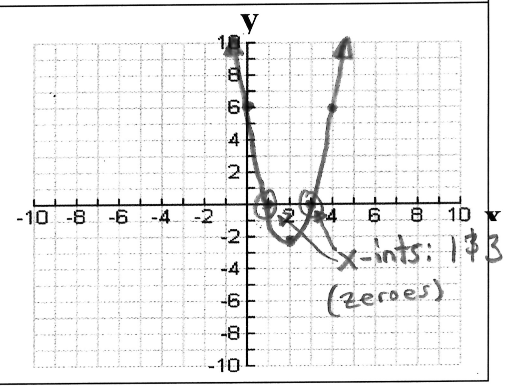
Seatwork: pg 204 # 2bdf, 8,14bc

Solving Polynomial Equations Using Technology

Minds On: Consider the following two problems:

Solve the equation $0 = 2x^2 - 8x + 6$	Graph the function $y = 2x^2 - 8x + 6$ and determine the x-intercepts.
$0 = 2(x^2-4x+3)$ $0 = 2(x-1)(x-3)$ $x=1$ or $x=3$ Solutions/ roots	$y = 2x^2-8x+6$ $y = 2(x^2-4x)+6$ $y = 2(x^2-4x+4-4)+6$ $y = 2(x^2-4x+4)+6-8$ $y = 2(x-2)(x-2)-2$ $y = 2(x-2)^2-2$ step pattern: 2, 6, 10 Vertex: $(2,-2)$ 

While these two tasks differ, it is important to note the similarities in each of their solutions. From this exercise, we see that to solve any one-variable equation we need only follow a simple four step algorithm:

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1. Move all terms to one side of the equal sign leaving a zero on one side.
2. Replace the zero with a y.
3. Graph the new function.
4. The x-intercepts are the solution to the original one variable problem.

Example 1

Use two methods to solve the equation $x^3 - 2x^2 = 33x - 90$.

Method 1 (Factor Theorem)

90
3 30
3 10
5 2

$\underbrace{x^3-2x^2-33x+90}_{f(x)}=0$

$f(3)=0$

$\therefore x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -33 & 90 \\ & & 3 & 3 & -90 \\ \hline & 1 & 1 & -30 & 0 \end{array}$$

$(x-3)(x^2+x-30)=0$

$(x-3)(x+6)(x-5)=0$

$x=3, -6, 5$

Method 2 (Graphing)

Move all terms to one side of the '=' sign.

$x^3-2x^2-33x+90=0$

Replace '0' with 'y'.

$y=x^3-2x^2-33x+90$

Graph the equation using technology.
What are the x-intercepts?

x-ints: $-6, 3, 5$

What is the solution to the original one variable equation

Soln $\rightarrow x=-6, 3, 5$

Example 2

Solve the equation $2x^3 - 4x^2 - 38x + 40 = 0$ using the factor theorem then verify your answer using graphing technology.

Method 1 (Factor Theorem)

$\underbrace{2(x^3 - 2x^2 - 19x + 20)}_{f(x)} = 0$

$f(1) = 0$

$\therefore x-1$ is a factor

$$\begin{array}{c|cccc} 1 & 1 & -2 & -19 & 20 \\ & & 1 & -1 & -20 \\ \hline & 1 & -1 & -20 & 0 \end{array}$$

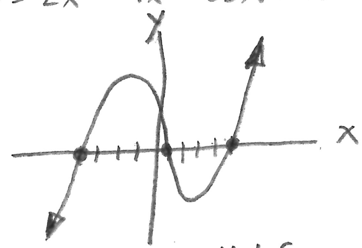
$2(x-1)(x^2 - x - 20) = 0$

$2(x-1)(x-5)(x+4) = 0$

$x = 1, 5, -4$

Method 2 (Graphing)

$y = 2x^3 - 4x^2 - 38x + 40$



x-ints: $-4, 1, 5$

$\therefore$ The sol$^{\text{ns}}$ $\rightarrow$ $x = -4, 1, 5$

Extension (Note: You will not be tested on this… but it is still highly valuable)

Although this unit focuses on solving polynomial equations, it is useful to note that this graphing technique can be used to solve any one variable equation.

Use graphing technology to solve the following:

1. a) $\cos(90t) = t$

$0 = t - \cos(90t)$

Graph $\rightarrow$ $y = t - \cos(90t)$

Sol$^{\text{n}}$ $\rightarrow$ $t \cong 0.595$

b) $2^x - x^2 + 9 = 0$

Graph $\rightarrow$ $y = 2^x - x^2 + 9$

Sol$^{\text{n}}$ $\rightarrow$ $x \cong -3.02$

2. The value of Mr. Ryan's investments are depreciating with respect to time:

$I = 20{,}000(0.91)^t$

Fortunately, he is employed and his bank account is increasing linearly:

$E = 10000 + 5000t$

- t is the time in years in both instances

a) How long will it take until Mr. Ryan's net worth is 50k?

Net Worth $= I + E$

$50000 = 20000(0.91)^t + 10000 + 5000t$

$0 = 20000(0.91)^t - 40000 + 5000t$

Graph $\rightarrow$ $y = 20000(0.91)^t - 40000 + 5000t$ $\rightarrow$ $t \cong 5.653$ years

b) How much faster would this happen if he stopped investing?

$N = 20000 + 10000 + 5000t$

$50000 = 20000 + 10000 + 5000t$

$\frac{20000}{5000} = \frac{5000t}{5000}$

$t = 4$ years

$\therefore$ If he stopped investing then he would reach his goal 1.653 years faster.