Hmwk: pg. 254 #1, 2odd, 3, 5odd, 6odd, 7a, 8ac, 9c, 10, 12

# Reciprocal Functions

## Practice

Determine the reciprocal of each value below:

a) $5 = \frac{5}{1}$ → $\frac{1}{5} = 0.2$

b) $10 = \frac{10}{1}$ → $\frac{1}{10} = 0.1$

c) $0.2 = \frac{1}{5}$ → $\frac{5}{1} = 5$

d) $0.5 = \frac{1}{2}$ → $\frac{2}{1} = 2$

e) $\infty = \frac{\infty}{1}$ → $\frac{1}{\infty}$ = a VERY small number

f) $-4 = -\frac{4}{1}$ → $-\frac{1}{4} = -0.25$

g) $-20 = \frac{-20}{1}$ → $-\frac{1}{20} = -0.05$

h) $-0.3 = -\frac{3}{10}$ → $-\frac{10}{3} = -3.\overline{3}$

i) $-0.1 = -\frac{1}{10}$ → $-\frac{10}{1} = -10$

j) $0 = \frac{0}{1}$ → $\frac{1}{0}$ = undefined

Note:

- The reciprocal values have the same sign as the original value.
- The reciprocal of positive values larger than one become values less than one.
- The reciprocal of positive values less than one become values greater than one.
- The reciprocal of zero is undefined.

## Example 1

If $f(x)$ is a function and $g(x) = \frac{1}{f(x)}$ then $g(x)$ is called the reciprocal function of $f(x)$.

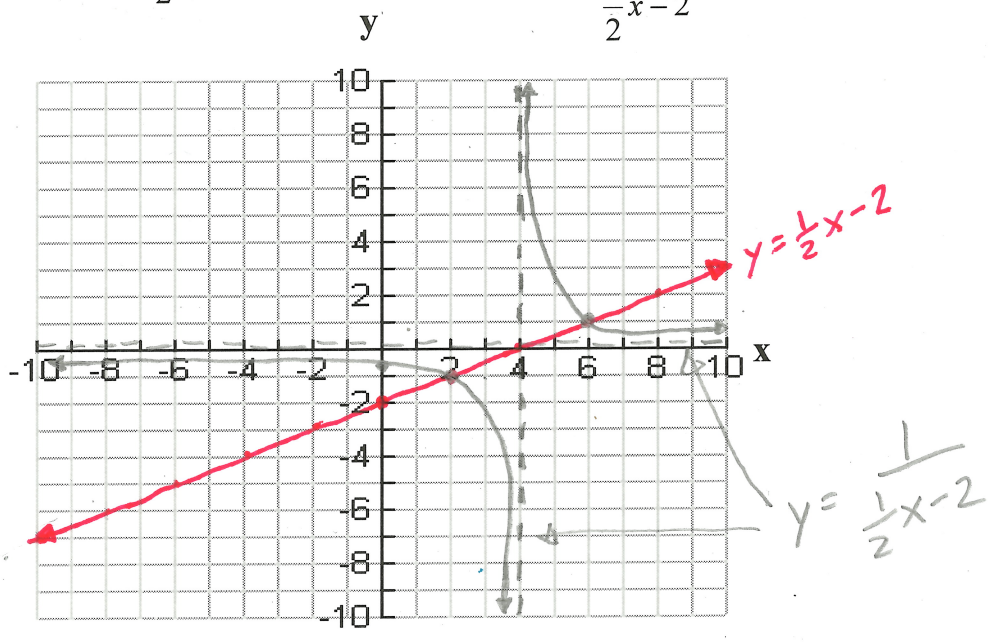
Graph the function $f(x) = \frac{1}{2}x - 2$ and its reciprocal $g(x) = \frac{1}{\frac{1}{2}x - 2}$.

1. V. Asymptotes
2. H. Asymptote
3. $y = \pm 1$

$f(x) = \frac{1}{2}x - 2$

$m = \frac{1}{2}$

$b = -2$



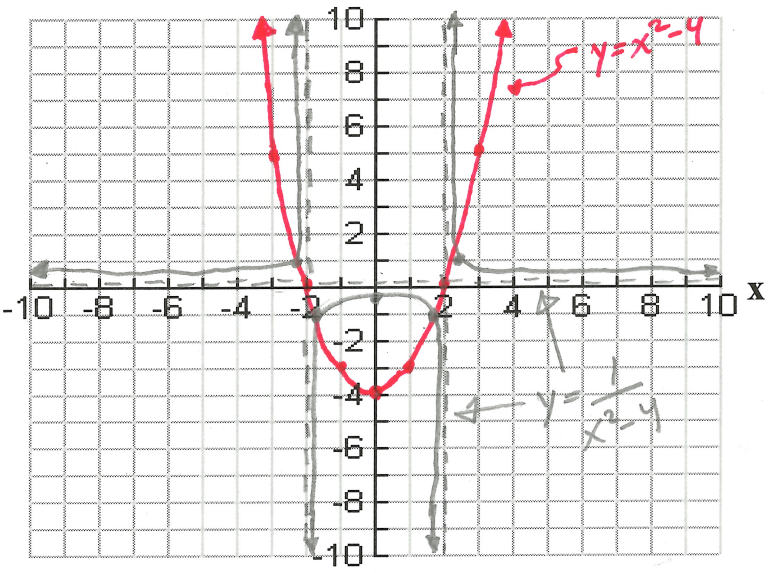
Example 2

Graph the following two functions on one Cartesian grid then complete the table below.

1. V. Asymptotes
2. H. Asymptote
3. $y = \pm 1$

Parent Function $f(x) = x^2 - 4$

Reciprocal Function $g(x) = \frac{1}{x^2 - 4}$




$f(x) = x^2 - 4$

vertex → (0, -4)

step pattern: 1, 3, 5

set $f(x) = 1$

$x^2 - 4 = 1$

$\sqrt{x^2} = \pm\sqrt{5}$

$x \cong \pm 2.24$

set $f(x) = -1$

$x^2 - 4 = -1$

$\sqrt{x^2} = \pm\sqrt{3}$

$x \cong \pm 1.73$

| Characteristic | $f(x) = x^2 - 4$ | $g(x) = \frac{1}{x^2 - 4}$ |
|---|---|---|
| x-intercepts/v. asymptotes | x-ints @ $x = \pm 2$ | V.A. @ $x = \pm 2$ |
| Positive Interval | $(-\infty, -2)$, $(2, \infty)$ | $(-\infty, -2)$, $(2, \infty)$ |
| Negative Interval | $(-2, 2)$ | $(-2, 2)$ |
| Increasing Interval | $(0, \infty)$ | $(-\infty, -2)$, $(-2, 0)$ |
| Decreasing Interval | $(-\infty, 0)$ | $(0, 2)$, $(2, \infty)$ |
| Points where y = 1 | $x \cong \pm 2.24$ | $x = \pm 2.24$ |
| Points where y = -1 | $x \cong \pm 1.73$ | $x = \pm 1.73$ |
| End Behaviours | as $x \to -\infty$, $y \to \infty$<br>as $x \to \infty$, $y \to \infty$ | as $x \to -\infty$, $y \to 0^+$<br>as $x \to \infty$, $y \to 0^+$ |

## Comparison of Graphed Functions and their Reciprocal

- x-intercepts on the parent function become vertical asymptotes on the reciprocal function.
- positive/negative intervals on the parent function remain +/- on the reciprocal function.
- increasing/decreasing intervals on the parent function are opposite on reciprocal functions.
- The points where y = 1 or y = -1 on the parent function are also points on the reciprocal function.
- If the degree of the denominator is larger than the numerator on the reciprocal function, then the function has a horizontal asymptote of y = 0.
- Where $0 < y < 1$ on the parent function, $y > 1$ on the reciprocal function and vice versa.
- Where $-1 < y < 0$ on the parent function, $y < -1$ on the reciprocal function and vice versa.

* Not 100% true