

Grade 11 Review – Rational Expressions

Warm-Up

Factor the following:

a) $x^2 - 4x - 32$
 $= (x - 8)(x + 4)$

$\begin{matrix} 32, 1 \\ 16, 2 \\ \textcircled{8, 4} \end{matrix}$

b) $4x^2 - 64$
 $= 4(x^2 - 16)$
 $= 4(x - 4)(x + 4)$

c) $3x^2y + 6xy^2$
 $= 3xy(x + 2y)$

d) $6x^2 + x - 2$ $P(-12)$ } $4, -3$
 $= 6x^2 + 4x - 3x - 2$ $S(1)$ }
 $= 2x(3x + 2) - 1(3x + 2)$
 $= (2x - 1)(3x + 2)$

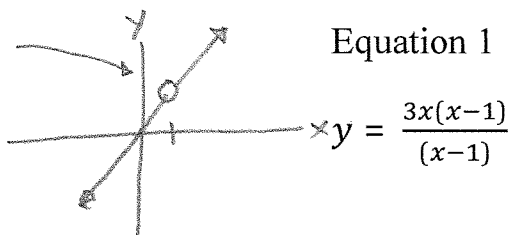
e) $2x^2 + 8x - 10$
 $= 2(x^2 + 4x - 5)$
 $= 2(x + 1)(x - 1)$

f) $x^2 - y^2 - 6y - 9$
 $= x^2 - (y^2 + 6y + 9)$
 $= x^2 - (y + 3)^2$ ← diff of squares
 $= [x - (y + 3)][x + (y + 3)]$
 $= (x - y - 3)(x + y + 3)$

Restrictions

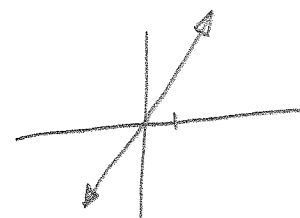
Consider the following two equations:

hole
@
 $x=1$



Equation 2

$y = 3x$



Are these two equations equivalent? Explain.

No! The graphs are similar but eqⁿ 1 has a restriction at $x=1$ and, therefore does not exist at that value

How can the two equations be made to be equivalent?

Add the restriction to eqⁿ 2 so it becomes $y = 3x, x \neq 1$

When a restriction is embedded into an expression it is referred to as an implicit restriction. When the restriction is written outside the expression it is referred to as an explicit restriction.

Example 1

State all restrictions (implicit and explicit) for each expression below:

$$a) \frac{2x}{x-4} + \frac{3}{x+3}, x \neq 5$$

$$x \neq 4, -3, 5$$

$$b) \frac{3x}{x^2-4} \times \frac{2y}{x-y}$$

$$= \frac{3x}{(x-2)(x+2)} \cdot \frac{2y}{x-y}$$

$$x \neq \pm 2$$

$$x \neq y$$

$$c) \frac{x}{x^2+6x+8} \div \frac{5x-3}{4}$$

$$= \frac{x}{(x+2)(x+4)} \div \frac{5x-3}{4}$$

$$x \neq -2, -4$$

$$\frac{5x-3}{4}$$

$$5x-3 \neq 0$$

$$\frac{5x \neq 3}{5 \quad 5}$$

$$x \neq \frac{3}{5}$$

When simplifying an expression, it is critically important to state all implicit restrictions that are removed through the simplifying process.

Example 2

Simplify the following rational expressions.

$$a) \frac{(x-1) \cdot 2}{x} + \frac{3}{(x-1)} \cdot x$$

$$= \frac{2x-2}{x(x-1)} + \frac{3x}{x(x-1)}$$

$$= \frac{5x-2}{x(x-1)}$$

$$b) \frac{15a}{a+1} \div \frac{5a}{a-2}$$

$$= \frac{15a}{a+1} \times \frac{a-2}{5a}$$

$$= \frac{3(a-2)}{a+1}, a \neq 0$$

$$-8) \begin{cases} P(-8) \\ S(-7) \end{cases}$$

$$c) \frac{m+5}{m^2-m-6} - \frac{2}{m-3}$$

$$= \frac{m+5}{(m-3)(m+2)} - \frac{2(m+2)}{(m-3)(m+2)}$$

$$= \frac{m+5-2m-4}{(m-3)(m+2)}$$

$$= \frac{-m+1}{(m-3)(m+2)}$$

$$d) \frac{x-1}{x^2-2x-8} \times \frac{2x^2-7x-4}{x^2-1}$$

$$= \frac{(x-1)}{(x-4)(x+2)} \cdot \frac{(2x+1)(x-4)}{(x+1)(x-1)}$$

$$= \frac{2x+1}{(x+2)(x+1)}, x \neq 1, 4$$

$$2x^2-7x-4$$

$$= 2x^2-8x+x-4$$

$$= 2x(x-4)+1(x-4)$$

$$= (2x+1)(x-4)$$

$$e) \frac{2x-2}{x^2+2x-3} - \frac{5}{x+3}$$

$$= \frac{2(x-1)}{(x+3)(x-1)} - \frac{5}{(x+3)}$$

$$= \frac{-3}{x+3}, x \neq 1$$

$$f) \frac{x+6}{x^2+3x-10} \div \frac{x-3}{2-x}$$

$$= \frac{x+6}{(x+5)(x-2)} \div \frac{-(x-2)}{x-3}$$

$$= \frac{-(x+6)}{(x+5)(x-3)}, x \neq 2$$

$$2-x$$

$$= -(-2+x)$$

$$= -(x-2)$$