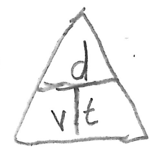
Hmwk: pg 303 # 1, 4, 5ac, 6ac, 7, 10ab, (13)

Rates of Change and Rational Functions

Example 1

Norbert, an avid sportsman, decides to canoe out to Math Island; the distance to this island is 2 km. Norbert can paddle his canoe at about 4m/s in calm water.

a) How long would it take Norbert to canoe out to the island and back in calm waters? Express your answer in minutes and seconds. Hint: $t = \frac{d}{v}$.



$t_{total} = t_1 + t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2}$

$= \frac{2000}{4} + \frac{2000}{4}$

$= 1000$ seconds

$= 16$ min 40 seconds

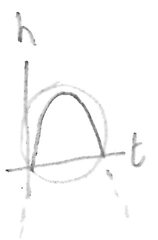
b) On a windy day, there is a current slowing Norbert down on his way to the island and speeding him up on the trip back to shore. Do you think that the trip will take the same amount of time, more time or less time? Create a hypothesis.

Most assume it will take the same time; the current slows him down on the way out but it speeds him up on the way back.

c) When Norbert canoes while there is a moving current his speed gets decreased/increased by an amount v_c. So the total travel time to the island and back is given by the equation:

$$t = \frac{2000}{4 - v_c} + \frac{2000}{4 + v_c}$$

Simplify the function and create a sketch.



$t = \frac{2000(4+V_c)}{(4-V_c)(4+V_c)} + \frac{2000(4-V_c)}{(4+V_c)(4-V_c)}$

$= \frac{8000 + \cancel{2000V_c} + 8000 - \cancel{2000V_c}}{(4-V_c)(4+V_c)}$

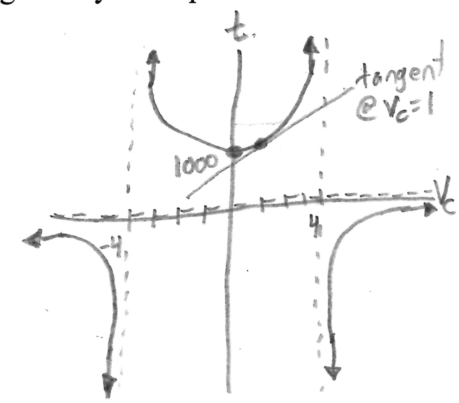
$t = \frac{16000}{(4-V_c)(4+V_c)}$

V.A. @ $V_c = \pm 4$

H.A. @ $t = 0$

V_c-int: None

t-int: 1000



d) In the context of the problem, why does the graph suggest that there is no value when v_c is equal to 4m/s?

If $V_c = 4$m/s then Norbert would be stuck on the shore and never make it out to Math Island; his net velocity would be 0 m/s.

e) What is the total travel time in minutes if the speed of the current is 1m/s?

set $V_c = 1$

$t(1) = \frac{16000}{(4-1)(4+1)} = 1066.\bar{6}$ seconds $\rightarrow$ 17 min 47 seconds

f) At what rate is the travel time, t, changing when the speed of the current is 1m/s?

$IROC \cong \frac{t(1.01) - t(1)}{0.01}$

$\cong \frac{1068.097918 - 1066.\bar{6}}{0.01}$

$\cong 143.1$ s/(m/s)

Example 2

The function $C(t) = \frac{5t}{0.01t^2+3.3}$ describes the concentration of a drug in the blood stream over time. The medication was taken orally at t = 0 minutes. The concentration, C, is measured in micrograms per millilitre (µg/ mL) and time, t, is measured in minutes.

a) Create a sketch of the function; hint… you may need graphing technology to assist.

$C(t) = \frac{5t}{0.01t^2+3.3}$ $\frac{+}{+}$

V.A./Holes: None

H.A. @ C = 0

t-int: 0

C-int: 0

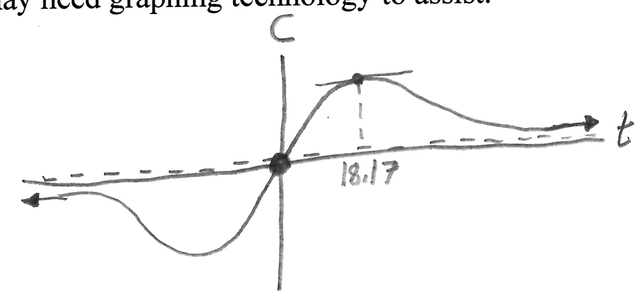
b^2-4ac

$=(0)^2-4(0.01)(3.3)$

$=-0.132$

Negative

∴ No V.A or holes



b) What is the concentration after 60 minutes?

$C(60) = \frac{5(60)}{0.01(60)^2+3.3}$

$= 7.63\mu g/mL$

c) Show that the maximum concentration occurs at t = 18.17 minutes by evaluating the IROC at that instant.

$IROC \doteq \frac{C(18.18)-C(18.17)}{0.01}$

$\simeq \frac{13.76204292-13.76204671}{0.01}$

$\cong -0.00038\ (\mu g/mL)/min$

$\cong 0(\mu g/mL)/min$

∴ Since the IROC $\cong$ 0(µg/mL)/min and the graph shows only a local max for stat. points where t > 0, there must be a local max @ t = 18.17 min.

d) At what rate is the concentration changing from 30 minutes to 60 minutes?

$AROC = \frac{C(60)-C(30)}{60-30}$

$= \frac{7.633587786-12.19512195}{30}$

$= -0.152(\mu g/mL)/min$