Hmwk: pg 235 #1, 2ai,b, e, 3, 4a, 5abd, 6abd, 8, 9, 12, (14), (15)

Rates of Change in Polynomial Functions

Recall:

$$\textit{Average Rate of Change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \qquad IROC \cong \frac{f(a+h) - f(a)}{h}$$

over the interval $x_1 \le x \le x_2$ at $x = a$, where $h \cong 0.01$

Example 1

a) Create a quartic function that has x-intercepts at x = -3, -1, 1, 3 and has a y-intercept of -5. Sketch the function.

$y = a(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

$y = a(x+3)(x+1)(x-1)(x-3)$

sub in $(0,-5)$

$-5 = a(3)(1)(-1)(-3)$

$\frac{-5}{9} = \frac{9a}{9}$

$a = -\frac{5}{9}$

lead term: $-\frac{5x^4}{9}$

$f(x) = -\frac{5}{9}(x+3)(x+1)(x-1)(x-3)$

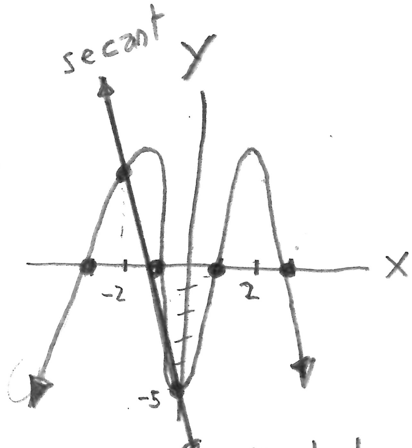
x-ints: $\pm 1, \pm 3$

y-int: -5

end behaviours:

as $x \to -\infty, y \to -\infty$

as $x \to \infty, y \to -\infty$



b) Is this an even or odd function? Explain your reasoning.

It has even symmetry since the graph is the same when reflected about the y-axis.

Also... $f(-x) = -\frac{5}{9}(-x+3)(-x+1)(-x-1)(-x-3)$

$= -\frac{5}{9}(-1)(x-3)(-1)(x-1)(-1)(x+1)(-1)(x+3)$

$= -\frac{5}{9}(x-3)(x-1)(x+1)(x+3)$

$\therefore f(-x) = f(x)$ which implies even symmetry.

c) On the interval from x = -2 to x = 0, do you expect the average rate of change to be positive or negative? Negative since the secant slopes downwards. Draw a secant on your graph and calculate the rate of change over that interval.

$$A.R.O.C. = \frac{f(0) - f(-2)}{(0) - (-2)}$$

$$= \frac{-5 - 8.\overline{3}}{2}$$

$$\cong -6.7$$

Example 2

Consider the function $f(x) = 2x^3 - 14x^2 + 30x - 18$. ← lead term

a) Create a sketch of this function.

$f(x) = 2(\underbrace{x^3 - 7x^2 + 15x - 9}_{g(x)})$

$g(1) = 0$

$\therefore x-1$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 15 & -9 \\ & & 1 & -6 & 9 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$f(x) = 2(x-1)(x^2-6x+9)$

$= 2(x-1)(x-3)(x-3)$

$= 2(x-1)(x-3)^2$

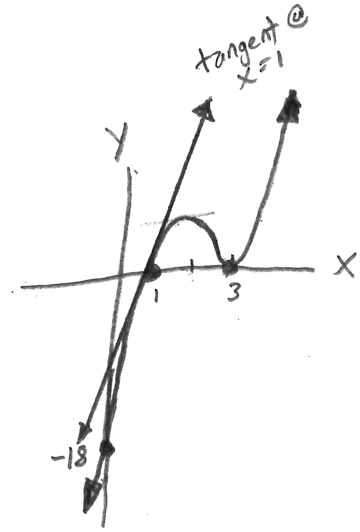
x-int: 1 and 3

y-int: −18

end behaviours

as $x \to -\infty$, $y \to -\infty$

as $x \to \infty$, $y \to \infty$



b) Draw a tangent at x = 1. Do you expect the instantaneous rate of change to be positive or negative at this point? Positive since the tangent slopes upwards.
Calculate the IROC at x = 1.

$$IROC \cong \frac{f(1.01) - f(1)}{0.01}$$

$$\cong \frac{0.079202 - 0}{0.01}$$

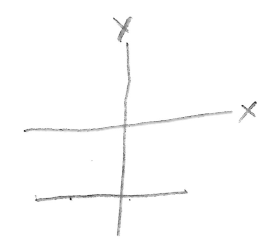
$$\cong 7.9$$

The Derivative (Extension)

The derivative is a function that is used to determine the IROC at any point on a graph. The derivative of a polynomial function is determined by following these steps:

1. Create a new coefficient by multiplying each coefficient by the exponent on x.
2. Drop each exponent by 1.

$y = -18$



Example 3

Determine the derivative of the function in example 2 and use it to verify your answer for the IROC when x = 1.

$y = 2x^3 - 14x^2 + 30x^1 - 18$

derivative (IROC) → $y' = 6x^2 - 28x^1 + 30$

set $x = 1$

$y' = 6(1)^2 - 28(1) + 30$

$= 6 - 28 + 30$

$= 8$

Bonus: Determine the coordinates of the local maximum by setting the derivative equal to zero and solving for x.

See attached!

Continued...

$y = 2x^3 - 14x^2 + 30x - 18$

$y' = 6x^2 - 28x + 30$

set $y' = 0$

$0 = 6x^2 - 28x + 30$

$0 = 2(3x^2 - 14x + 15)$ $\left.\begin{matrix} P(45) \\ S(-14) \end{matrix}\right\} -9, -5$

$0 = 2(3x^2 - 9x - 5x + 15)$

$0 = 2[3x(x-3) - 5(x-3)]$

$0 = 2(3x-5)(x-3)$

$x = \frac{5}{3}$ or $x = 3$

(~ 1.67)

Δ local max

$x = \frac{5}{3}$

$y = 2(5/3)^3 - 14(\frac{5}{3})^2 + 30(\frac{5}{3}) - 18$

$y \cong 2.37$

$\rightarrow (1.67, 2.37)$