Rates of Change in Exponential and Logarithmic Functions

1. Mr. Ryan refuses to update the iOS on his apple product. Each year, due to compatibility issues, the number of apps that he can use decreases exponentially as shown in the table below.

Year (t)	Number of Compatible Apps (1000s) (A)
2011	1500
2012	1200
2013	960
2014	768
2015	614
2016	492
2017	393
2018	315
2019	252
2020	201

iOS 5

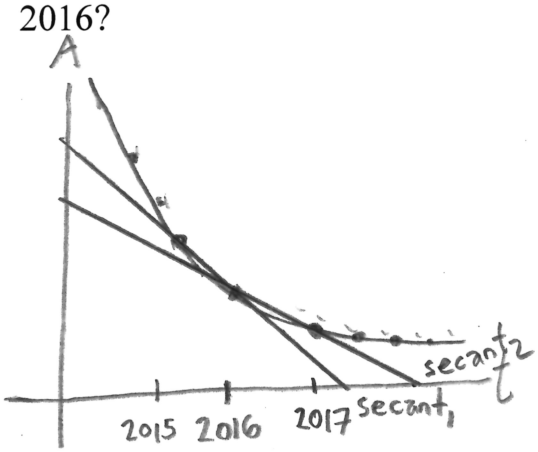
a) What was the average rate of change in number of compatible apps from 2011 to 2015?

$$AROC = \frac{\Delta A}{\Delta t} = \frac{A(2015)-A(2011)}{2015-2011} = \frac{614-1500}{4} = -221.5 \text{ thousand apps/year}$$

b) What was the average rate of change in number of compatible apps from 2016 to 2020?

$$AROC = \frac{\Delta A}{\Delta t} = \frac{A(2020)-A(2016)}{2020-2016} = \frac{201000-492000}{4} = -72250 \text{ apps/year}$$

c) Using the table above, how could you estimate the instantaneous rate of change for the year 2016?



$$AROC_1 = \frac{A(2016)-A(2015)}{2016-2015} = \frac{492-614}{1} = -122000 \text{ apps/year}$$

$$AROC_2 = \frac{A(2017)-A(2016)}{2017-2016} = \frac{393-492}{1} = -99000 \text{ apps/year}$$

$$IROC \approx \frac{AROC_1 + AROC_2}{2} \approx \frac{(-122000)+(-99000)}{2}$$

$$\boxed{= -110500 \text{ apps/year}}$$

2. Mr. Ryan used to drive an ugly car. The beautiless set of wheels was purchased in the year 2003 for \$12000. The value of the car, V, depreciates according to the following formula:

$V = 12000(0.88)^t$

- where t is the time in years since 2003

a) How much would the ugly car be worth today?

set $t = 17$ years

$V = 12000(0.88)^{17}$

$V = \$1365.80$

b) At what rate is the car depreciating in value in (\$/year) at the time when his ugly car is worth \$1000?

set $V = 1000$

$\frac{1000}{12000} = \frac{\cancel{12000}(0.88)^t}{\cancel{12000}}$

$\frac{1}{12} = 0.88^t$

$t = \log_{0.88} \frac{1}{12}$

$t = \frac{\log \frac{1}{12}}{\log 0.88}$

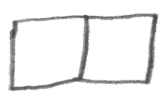
$t \doteq 19.44$ years

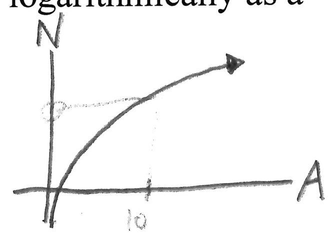
$IROC \cong \frac{V(19.45) - V(19.44)}{0.01}$

$\cong \frac{998.5486282 - 999.8259228}{0.01}$

$\cong \$ -127.73/\text{year}$

3. The number of plant species found in a patch of lawn, N, increases logarithmically as a function of the area of grass being inspected as follows:





$N = 10.7\log A + 8.1$

where

- A is the area of the lawn being inspected (m^2)

a) How much grass needs to be inspected to find 30 plant species?

set $N = 30$

$30 = 10.7\log A + 8.1$

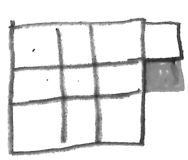
$\frac{21.9}{10.7} = \frac{\cancel{10.7}\log A}{\cancel{10.7}}$

$\frac{21.9}{10.7} = \log A$

$A = 10^{\frac{21.9}{10.7}}$

$A = 111.4 m^2$

b) At what rate is the number of species increasing when inspecting only 10 m^2 of grass?



$IROC \doteq \frac{N(10.01) - N(10)}{0.01}$

$\cong \frac{18.80464463 - 18.8}{0.01}$

$= 0.464$ species$/m^2$