Hmwk: pg 369 # 1,2, 3, 4ac, 5, 6, 7a, 8, 9ab, 12, 14*, (15)

Rates of Change with Trigonometric Functions

Consider Renee DesCartes ride on the Ferris wheel from the previous lesson.

- He boards the ride 2 m above the ground.
- The Ferris wheel has a diameter of 30 m.
- The wheel turns one full revolution every 5 minutes.

The equation that described this scenario was: $H(t) = -15\cos\left[\frac{2\pi t}{5}\right] + 17$

If we represent the time in seconds instead of minutes, the 'k' value would be:

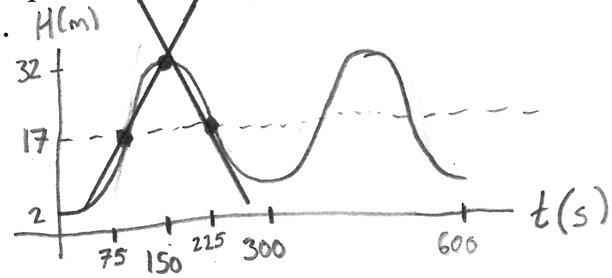
$$k = \frac{2\pi}{300} = \frac{\pi}{150}$$

The modified equation now becomes: $H(t) = -15\cos\left[\frac{\pi t}{150}\right] + 17$

Follow-Up

secant₂ secant₁

1. Sketch a graph to represent the height of Descartes' Ferris wheel ride with time measured in seconds.



2. Compare the secants for the intervals $75 \le t \le 150$ and $150 \le t \le 225$.

a) What would you expect to be that same for the rates of change over these intervals? Magnitude

b) What would you expect to be different about the rates of change for these intervals? Secant$_1$ → positive AROC

Secant$_2$ → negative AROC

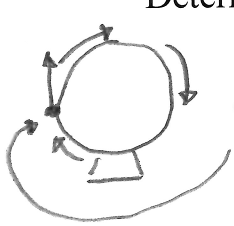
c) Verify your answers in parts a) and b) by calculating the relevant AROC.

$$AROC_1 = \frac{H(150)-H(75)}{150-75} = \frac{32-17}{75} = 0.2\,m/s$$

$$AROC_2 = \frac{H(225)-H(150)}{225-150} = \frac{17-32}{75} = -0.2\,m/s$$

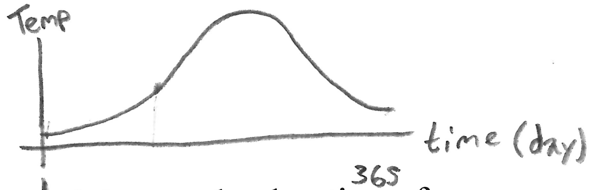
3. When do you think that ~~the~~ Renee's height is increasing at the greatest rate? 75 seconds

Determine the rate of change at that instant?



greatest increase in height @ t=75s.

$$IROC \cong \frac{H(75.01)-H(75)}{0.01} \cong \frac{17.00314159-17}{0.01} \cong 0.314\,m/s$$



Temperature Trends

The average monthly temperature in Guelph over the duration of a year can be seen in the following table:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Avg. Temp.	-7.4	-5.9	-1.2	5.1	11.9	17.9	19.6	18.9	14.3	8	1.5	-3.6

If we were to fit a sinusoidal regression through these points to represent the temperature as a function of day number (Jan 1st = 1, Dec 31st = 365), we get the following function:

$$Temp(day) = 13.6\,sin(0.01674day - 1.83877) + 6.3$$

1. According to the equation above, what is the period in days for one cycle of temperature changes?

$T = \frac{2\pi}{|k|} = \frac{2\pi}{0.01674} = 375$ days → somewhat close to a year (365 days)

2. What day of the year is today? What is the predicted temperature for today?

$\boxed{\text{Nov } 24^{th}}$

$day = 31+28+31+30+31+30+31+31+30+31+24$

$day = 328$

$Temp(328) = 13.6 \sin[0.01674(328) - 1.83877] + 6.3 = -0.343451594°C$

3. What is the rate of change for today?

$IROC \cong \frac{Temp(328.01) - Temp(328)}{0.01}$

$\cong \frac{(-0.345438029) - (-0.343451594)}{0.01} \cong -0.19864°C/day$

4. Instantaneous rates of change are great for making short term predictions. What do you expect the temperature will be tomorrow?

$Temp_{329} = Temp_{328} + \Delta days \times IROC$

$= -0.34345°C + (1)(-0.19864°C/day)$

$\cong -0.54209°C$

5. Calculate the temperature for tomorrow using the formula. Is this answer close to the answer you got in question 4?

$Temp(329) = 13.6 \sin[0.01674(329) - 1.83877] + 6.3$

$\cong -0.54116°C$

The answer is very close!

6. Use the IROC you calculated in question 3 to predict the average temperature 100 days from now. Compare this answer to that which is given to you by using the formula. What can you say about using IROC's for long term predictions?

$Temp_{428} = Temp_{328} + \Delta days \times IROC$

$= -0.343451594 + (100)(-0.19864)$

$\cong -20.2°C$

$Temp(428) = 13.6 \sin[0.01674(428) - 1.83877] + 6.3$

$\cong -4.8°C$

The IROC is helpful for short term predictions but not as useful for longer terms.