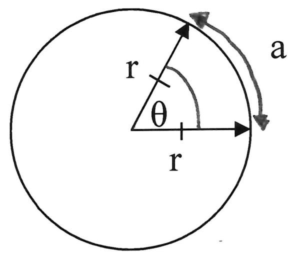
Hmwk: pg 330#1-8(odd), 9, 10, (11), 13

Radian Measure and Special Triangles

Recall:

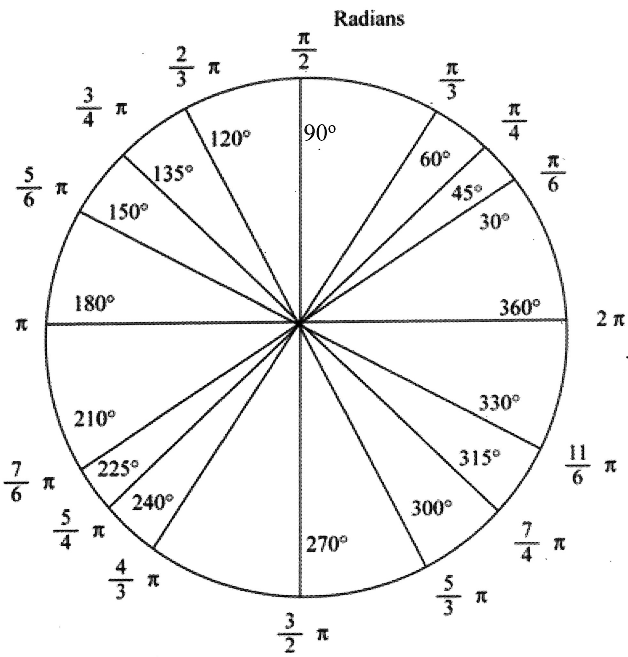
An angular measure 'θ', in radians, is the ratio of arc length 'a' to the length of the radial arm 'r'.

$$\theta(in\ radians) = \frac{a}{r}$$



$$180^o = \pi\ radians$$

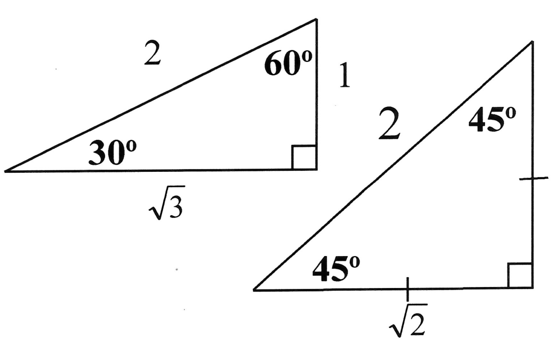
On a unit circle, some of the angular measures in degrees and radians appear as follows.



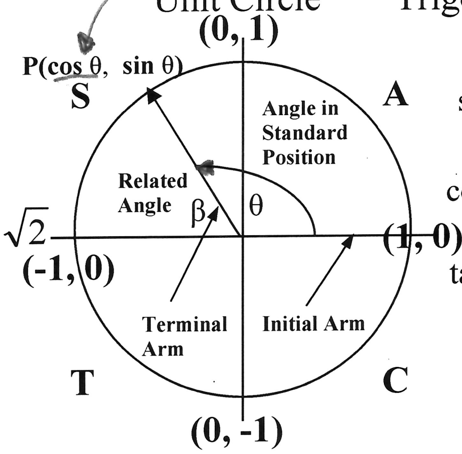
$\frac{\cos\theta}{1} = \frac{x}{r}$ $x = r\cos\theta$ if $r=1$ $\boxed{x=\cos\theta}$

Grade 11 Cheat Sheet for Trigonometry

Special Triangles



Unit Circle



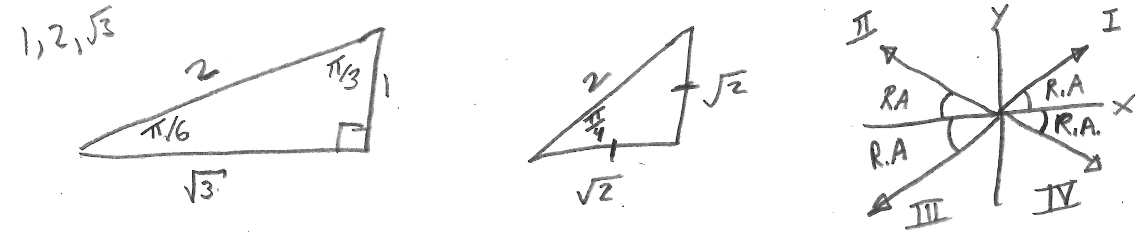
Trigonometric Ratios

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

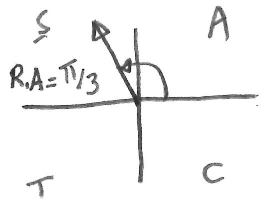
$\boxed{\text{SYR CXR TYX}}$



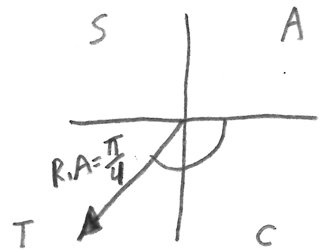
Example 1

Use a sketch and the CAST rule to determine the related angle and sign of each trigonometric ratio.

a) $\cos\left(\frac{2\pi}{3}\right)$ sign → ⊖

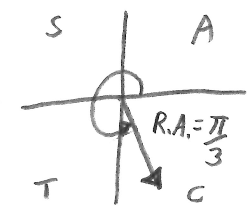


b) $\tan\left(-\frac{3\pi}{4}\right)$ sign → ⊕



c) $sec\left(\frac{5\pi}{3}\right)$ sign → ⊕

$= \dfrac{1}{\cos\left(\frac{5\pi}{3}\right)}$



Example 2

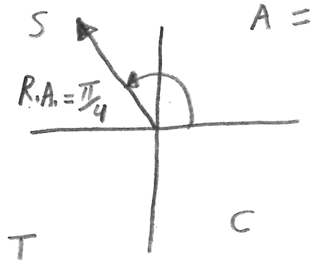
Evaluate the exact value for each of the following.

a) $\sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{3\pi}{4}\right)$ $= -\cos\left(\frac{\pi}{4}\right)$

$A = -\frac{\sqrt{2}}{2}$



c) $\cot\left(\frac{7\pi}{6}\right)$

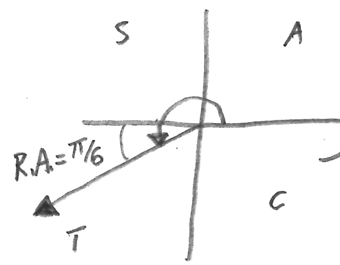
$= \dfrac{1}{\tan\left(\frac{7\pi}{6}\right)}$ → $= \dfrac{1}{\tan\left(\frac{\pi}{6}\right)}$

$= \dfrac{1}{1/\sqrt{3}}$

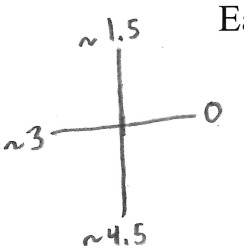
$= 1 \div \frac{1}{\sqrt{3}}$

$= 1 \times \frac{\sqrt{3}}{1}$

$= \sqrt{3}$



Example 3

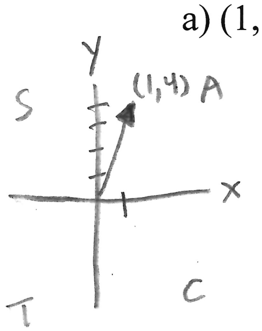


Each point below lies on the end of a terminal arm.

- Draw a sketch to represent the scenario.
- Determine the value of r using $x^2 + y^2 = r^2$.
- Determine one of the primary trig ratios.
- Calculate the radian value of the angle in standard position θ.

a) (1, 4)

x y



$r^2 = x^2 + y^2$

$r^2 = (1)^2 + (4)^2$

$\sqrt{r^2} = \sqrt{17}$

$r = \sqrt{17}$

$\sin\theta = \frac{y}{r}$

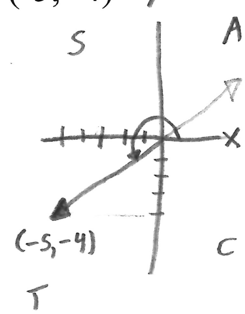
$\sin\theta = \frac{4}{\sqrt{17}}$

$\theta = \sin^{-1}\left(\frac{4}{\sqrt{17}}\right)$

$\theta \cong 1.326$

b) (-5, -4)

x y



$r^2 = x^2 + y^2$

$r^2 = (-5)^2 + (-4)^2$

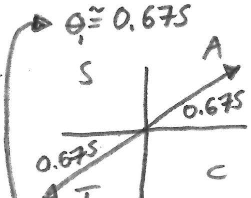
$\sqrt{r^2} = \sqrt{41}$

$r = \sqrt{41}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-4}{-5}$

$\theta = \tan^{-1}\left(\frac{-4}{-5}\right)$ → $\theta \cong 0.675$



$\theta_2 = \pi + 0.675$

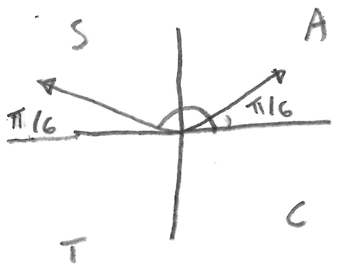
$\cong 3.816$

Example 4

Solve for the angle θ; $0 \le \theta \le 2\pi$. Use special triangles if applicable.

a) $\sin\theta = \frac{1}{2}$

R.A. = $\pi/6$

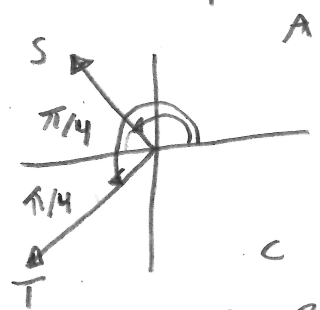


$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \frac{5\pi}{6}$

b) $\cos\theta = -\frac{\sqrt{2}}{2}$

R.A. = $\frac{\pi}{4}$



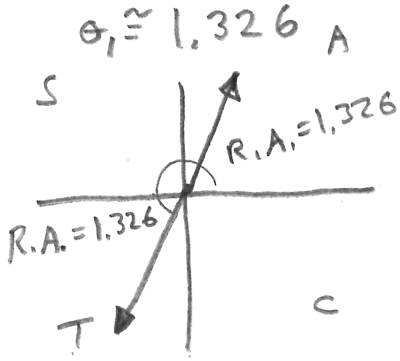
$\theta_1 = \pi - \pi/4 = \frac{3\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

c) $\tan\theta = 4$

$\theta = \tan^{-1}(4)$

$\theta_1 \cong 1.326$



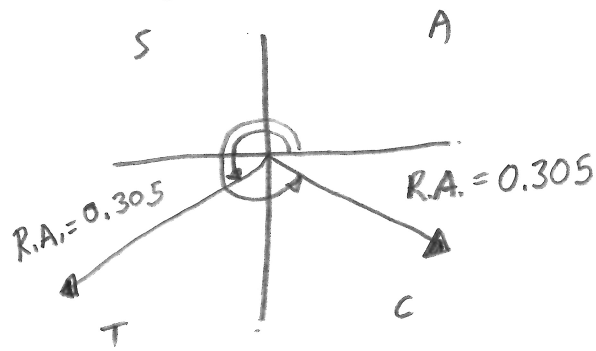
$\theta_2 = \pi + 1.326$

$\theta_2 \cong 4.467$

d) $\sin\theta = -0.3$

$\theta = \sin^{-1}(-0.3)$

$\theta \cong -0.305$



$\theta_1 \cong \pi + 0.305$ $\cong 3.446$

$\theta_2 \cong 2\pi - 0.305$ $\cong 5.978$