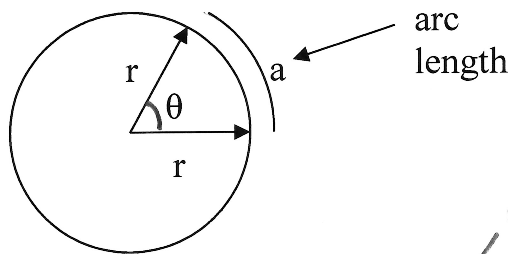
Homework: pg 320 #1ace, 3ac, 4ac, 5, 6, 7- 8(ace), 9, 11, 12

Radian Measure

Consider the following diagram below:



fraction

A radian is a unit of angular measure. It is calculated as the ratio of arc length used to make the curved side to the length of the radial arm. That is…

$$\boxed{\theta\,(in\ radians) = \frac{a}{r}}$$

Since both the arc length and radial arm are measured in the same units of length, an angular measure in radians is unitless. Consequently, any angle that is written without units is assumed to be in radians by default.

If an angular measure was taken to represent a full rotation (360°), then the arc length would be equal to the circumference of a circle. In that scenario,

$a = 2\pi r$ (for one full angular rotation)

If we substitute this into the above formula for angular measure, we get:

$$\theta = \frac{a}{r}$$

$$\theta = \frac{2\pi r}{r}$$

$$\theta = 2\pi$$

(radians) equivalent to 360°

From this result, we can conclude the following relationship between an angle measured in degrees and the angle measure in radians.

$$2\pi\ radians = 360^o$$

or

$$\boxed{\pi\ radians = 180^o}$$

Examples

$\frac{\pi}{3}$ rads $= 60°$

$30° = \frac{\pi}{6}$ rads

The following pair of equations can be used to convert angular measures from one unit of measure to the other:

$$angle(in\ radians) = angle(in\ ^{o}) \times \frac{\pi}{180^{o}}$$

$$angle(in\ ^{o}) = angle(in\ radians) \times \frac{180^{o}}{\pi}$$

Example 1

a) Convert each angular measure to radians.

i) $90^0 = \frac{\cancel{90^0}}{1} \times \frac{\pi}{180^{\cancel{0}}} = \frac{90\pi}{180} = \frac{\pi}{2}$

ii) $120^0 = \frac{120^{\cancel{0}}}{1} \times \frac{\pi}{180^{\cancel{0}}} = \frac{120\pi}{180} = \frac{2\pi}{3}$ radians

iii) $30^0 = \frac{30^\circ}{1} \times \frac{\pi}{180^\circ} = \frac{30\pi}{180} = \frac{\pi}{6}$ rads

b) Convert each angular measure to degrees.

i) $\frac{\pi}{4} = \frac{\cancel{\pi}}{4} \times \frac{180^\circ}{\cancel{\pi}} = \frac{180^\circ}{4} = 45^\circ$

ii) $-\frac{5\pi}{4} = -\frac{5\cancel{\pi}}{4} \times \frac{180^\circ}{\cancel{\pi}} = \frac{-900^\circ}{4} = -225^\circ$

iii) $5 = \frac{5}{1} \times \frac{180^\circ}{\pi} = \frac{900^\circ}{\pi} \cong 286.48^\circ$

Example 2

A bicycle wheel with a diameter of 40 cm makes 10 revolutions per second.

a) Determine the angular velocity of the wheel in radians per second.

angular velocity

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{10 \times 2\pi}{1s} = 20\pi \text{ rads/s}$$

b) Determine how far the bike has travelled in 2 minutes.

$d = vt$

$\theta = \omega t$

$\theta = (20\pi \text{ rads/s})(120 s)$

$= 2400\pi$ rads

$\frac{\theta}{1} = \frac{a}{r}$

$a = r\theta$

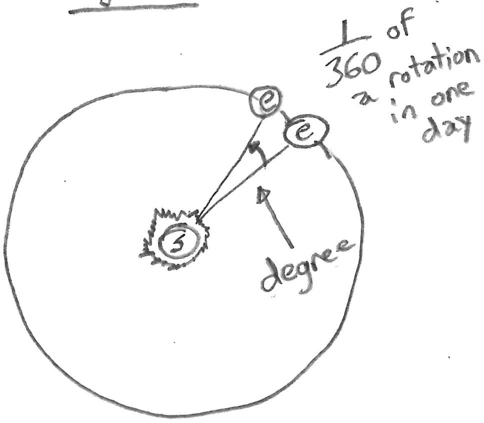
distance $= a = r\theta$

$= (20cm)(2400\pi)$

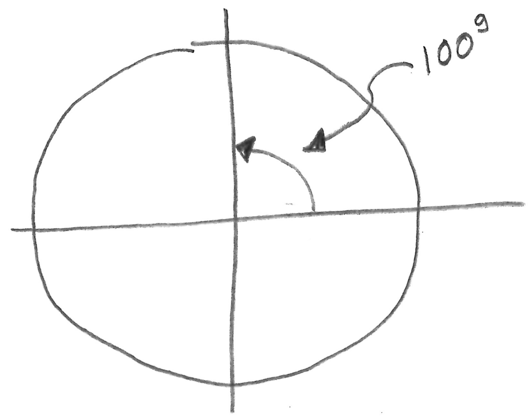
$= 48000\pi$ cm

$\cong 1.51$ km

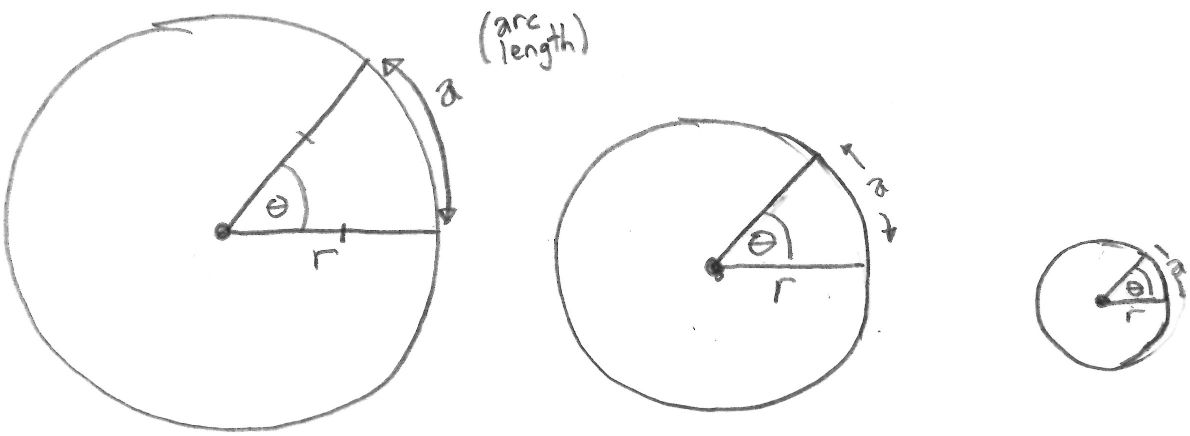
Degrees



Gradians



Radians



$\theta = \frac{a}{r}$ (radians)

$= \frac{3\cancel{cm}}{3\cancel{cm}}$

$= 1$ rad

$\theta = \frac{a}{r}$

$= \frac{2cm}{2cm}$

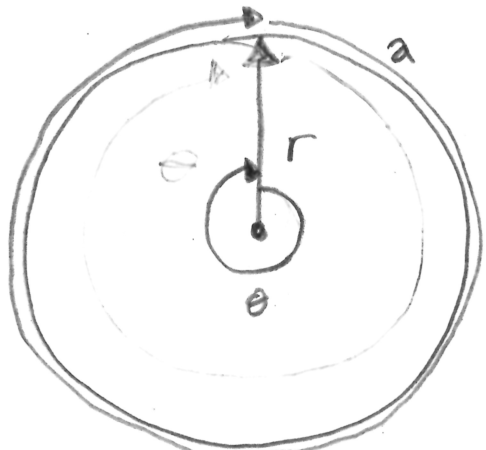
$= 1$ rad

$\theta = \frac{a}{r}$

$= \frac{0.8\cancel{cm}}{0.8\cancel{cm}}$

$= 1$ rad

One full rotation



$$\boxed{\underset{\text{(radians)}}{\theta} = \frac{a}{r}} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi \text{ radians}$$

arc length = circumference
$= 2\pi r$

$$\boxed{2\pi \text{ radians} = 360^\circ}$$

π radians $= 180^\circ$