Grade 11 Review – Quadratic Functions

A quadratic function can be expressed in three forms; vertex, standard and factored. Each form provides valuable information about the graph of the function.

Example 1

For each quadratic relation below, determine the x-intercepts, y-intercepts, and vertex then graph the parabola.

a) $y = x^2 - 6x + 5$ standard form

$y = (x-1)(x-5)$ factored form

step pattern: 1, 3, 5

x-ints: 1 and 5

y-int: 5

Vertex

$x = \frac{1+5}{2}$

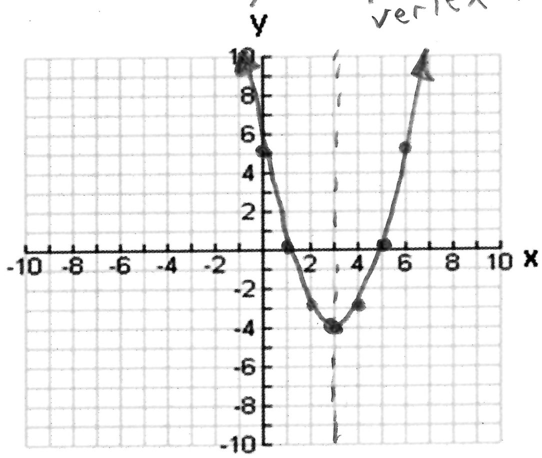
$x = 3$

$y = (3-1)(3-5)$

$= (2)(-2)$

$y = -4$

vertex → (3, -4)



b) $y = -2x^2 - 12x - 16$

$y = -2(x^2 + 6x + 8)$

$y = -2(x+2)(x+4)$

x-ints: -2 and -4

y-int: -16

step pattern: -2, -6, -10

$y = -2x^2 - 12x - 16$

$y = -2(x^2 + 6x) - 16$

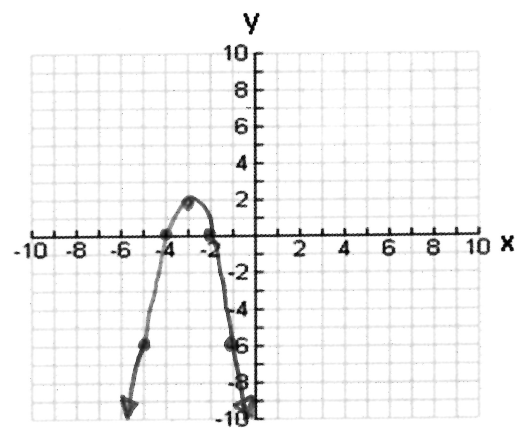
$y = -2(x^2 + 6x + 9 - 9) - 16$

$y = -2(x^2 + 6x + 9) + 18 - 16$

$y = -2(x+3)(x+3) + 2$

$y = -2(x+3)^2 + 2$

vertex → (-3, 2)



Example 2

Create a quadratic function that passes through the point (2, -10) and has a vertex (-1, 8).

$y = a(x-h)^2 + k$

$y = a(x+1)^2 + 8$

Sub in (2, -10)

$-10 = a(2+1)^2 + 8$

$-10 = 9a + 8$

$\frac{-18}{9} = \frac{9a}{9}$

$a = -2$

$\boxed{y = -2(x+1)^2 + 8}$

Example 3

Create a quadratic function that passes through the point (8, 21) and has x-ints of 1 and 7.

$y = a(x - x_1)(x - x_2)$

$y = a(x-1)(x-7)$

Sub in (8, 21)

$21 = a(8-1)(8-7)$

$\frac{21}{7} = \frac{7a}{7}$

$a = 3$

$\boxed{y = 3(x-1)(x-7)}$

The discriminant ($b^2 - 4ac$) can be used to determine the number of roots of a quadratic equation.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Example 4

Determine the number of solutions for each quadratic equation; do not solve.

a) $2x^2 - 20x + 50 = 0$

$b^2-4ac = (-20)^2 - 4(2)(50)$
$= 400 - 400$
$= 0$ $\therefore$ One real solution

b) $x^2 = -2x - 3 \rightarrow x^2+2x+3=0$

$b^2-4ac = (2)^2 - 4(1)(3)$
$= 4 - 12$
$= -8 \leftarrow$ negative

$\therefore$ No real solutions

Quadratic equations can be solved by factoring or using the quadratic formula

Example 5

Solve the following quadratic equation using both methods.

$6x^2 + 7x - 5 = 0$

Method 1 (Factoring)

$P(-30)\ 10, -3$
$S(7)$

$6x^2+7x-5=0$
$6x^2+10x-3x-5=0$
$2x(3x+5)-1(3x+5)=0$
$(2x-1)(3x+5)=0$

$2x-1=0$
$\frac{2x}{2}=\frac{1}{2}$
$x=\frac{1}{2}$

$3x+5=0$
$\frac{3x}{3}=\frac{-5}{3}$
$x=-\frac{5}{3}$

Method 2 (Q. Formula)

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2-4(6)(-5)}}{2(6)}$

$x = \frac{-7 \pm 13}{12}$

$x = \frac{6}{12} = \frac{1}{2}$ $x = \frac{-20}{12} = \frac{-5}{3}$

Grade 11 Review – Exponential Functions

Rational exponents are evaluated by separating the exponent into a power and root.

Example 6

$x^{-n} = \frac{1}{x^n}$

Evaluate the following.

a) $4^{\frac{5}{2}}$

$= (\sqrt[2]{4})^5$
$= (2)^5$
$= 32$

or $= \sqrt[2]{4^5}$
$= \sqrt[2]{1024}$
$= 32$

b) $27^{-\frac{4}{3}}$

$= \frac{1}{27^{4/3}}$
$= \frac{1}{(\sqrt[3]{27})^4}$
$= \frac{1}{3^4}$
$= \frac{1}{81}$

c) $(16x^8)^{\frac{3}{4}}$

$= 16^{3/4}(x^8)^{3/4}$
$= (\sqrt[4]{16})^3\ x^{24/4}$
$= (2)^3 x^6$
$= 8x^6$

Exponential equations are solved by finding a common base or using trial and error.

Example 7

Solve each exponential equation

a) $4^{2x-1} = 8^{x+3}$

$(2^2)^{(2x-1)} = (2^3)^{(x+3)}$
$2^{4x-2} = 2^{3x+9}$
$4x-2 = 3x+9$
$x = 11$

b) $2^{10x-1} = \frac{1}{16}$

$2^{10x-1} = 2^{-4}$
$10x-1 = -4$
$\frac{10x}{10} = \frac{-3}{10}$
$x = -\frac{3}{10}$

c) $3^x = 15$

$\log 3^x = \log 15$
$\frac{x \log 3}{\log 3} = \frac{\log 15}{\log 3}$
$x \doteq 2.46$

Exponential functions can be used to model real world scenarios such as compound interest investments, general exponential growth and decay problems or half-life scenarios.

$$A = P(1+i)^n \qquad y = a(1 \pm r)^x \qquad y = a\left(\frac{1}{2}\right)^{\frac{t}{h}} \qquad y = a(2)^{t/d}$$

Example 8

a) Shalen invests $10000 at 6% compounded semi-annually. How long will it take until the investment is worth $20000? $A = P(1+i)^n$

$A = 20000$
$P = 10000$
$i = 0.06 \div 2 = 0.03$
$n = ?$

$\frac{20000}{10000} = \frac{10000(1.03)^n}{10000}$

$2 = 1.03^n$

$\log 2 = \log 1.03^n$

$\frac{\log 2}{\log 1.03} = \frac{n \log 1.03}{\log 1.03}$

$n \doteq 23.4$ half years

$\doteq 11.7$ years

b) Ben purchases a car for $40000 that depreciates by 16% each year. How much will the car be worth in a decade? $y = a(1-r)^t$

$y = 40000(1-0.16)^t$

$y = 40000(0.84)^t$

set $t = 10$

$y = 40000(0.84)^{10}$

$y \doteq \$6996.05$

Practice
1. Determine the x-intercepts, y-intercepts and vertex of the quadratic function $y = -2x^2 + 4x + 6$ then sketch the function.
2. Determine the equation of a quadratic function that has a vertex of (4, 8) and an x-intercept of 6.
3. Determine the equation of a quadratic function that has a y-int of -27 and has x-intercepts of 3 and -3.
4. How many solutions does each quadratic equation have?
a) $x^2 - 8x + 10 = 0$ b) $x^2 + 4x + 10 = 0$
5. Solve the following quadratic equations.
a) $2x^2 + 7x + 3 = 0$ b) $x^2 + 5x + 7 = 0$
6. Evaluate each power.
a) $16^{\frac{3}{2}}$ b) $125^{-\frac{2}{3}}$
7. Evaluate the following.
a) $6^{3x-5} = 36^{x+3}$ b) $5^x = 1050$
8. The amount of bacteria in a Petrie dish grows exponentially with time. If there is initially 200 cells in the dish and the amount grows by 25% every hour, how long will it take until there is 10000 cells?

Answers
1. x-ints: -1, 3, y-int: 6, vertex: (1, 8), 2. $y = -2(x - 4)^2 + 8$, 3. $y = 3(x - 3)(x + 3)$
4.a)2, b) none , 5.a) x = -1/2 or -3, b) No Sol^n, 6.a) 64, b) 1/25 (0.04),
7.a) x = 11, b) x = 4.32, 8. 17.5 hours