Hmwk: pg 417 # 5ac, 8, 9ac, 10ace, 11cegijl, 16a

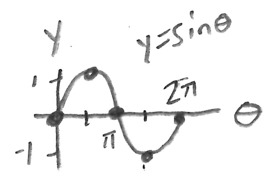
Proving Trigonometric Identities: Part 1

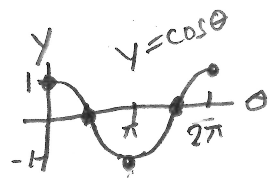
Trigonometric identities can be proven true or untrue by graphing; this method is typically convenient if you have access to graphing technology.

Example 1

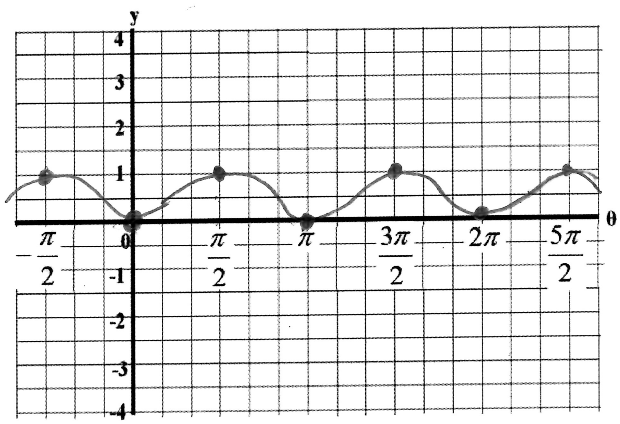
Prove that the following identity is true by graphing.

$\sin^2\theta = 1 - \cos^2\theta$

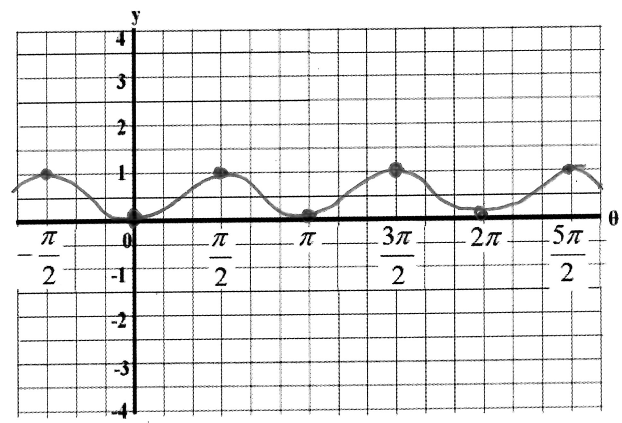




Graph $y = \sin^2\theta$.



Graph $y = 1 - \cos^2\theta$



Same graphs... L.S. = R.S. QED

Example 2

Identities can be disproven by finding any single value that makes the mathematical statement false. Show that the following statement is false.

$\sin\theta = \cos\theta$

$2^x = x^2$ ← sub in $x = 2$

L.S. = 4 R.S. = 4

Choose an arbitrary value for θ and evaluate each side.

Let $\theta = \frac{\pi}{4}$.

L.S. $= \sin\left(\frac{\pi}{4}\right)$ $= \frac{\sqrt{2}}{2}$

R.S. $= \cos\left(\frac{\pi}{4}\right)$ $= \frac{\sqrt{2}}{2}$

L.S. = R.S. for $\theta = \frac{\pi}{4}$

Let $\theta = 0$.

L.S. $= \sin(0)$ $= 0$

R.S. $= \cos(0)$ $= 1$

L.S. ≠ R.S.

∴ This identity is not true.

Example 3

Trigonometric identities can be proven true algebraically by making one side of the equation look like the other.
Note: Keep left side and right side separated.

Prove the following:

a) $\dfrac{\cos(x-y)}{\cos(x+y)} = \dfrac{1+\tan x \tan y}{1-\tan x \tan y}$

Hint: It is often strategic to change tanθ to sinθ and cosθ using the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

$$\text{L.S.} = \frac{\cos(x-y)}{\cos(x+y)}$$

$$\text{R.S.} = \frac{1+\tan x\tan y}{1-\tan x\tan y}$$

$$= \frac{1+\frac{\sin x \sin y}{\cos x \cos y}}{1-\frac{\sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\frac{\cos x \cos y}{\cos x \cos y}+\frac{\sin x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y}{\cos x \cos y}-\frac{\sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\cos x\cos y+\sin x\sin y}{\cos x\cos y} \div \frac{\cos x\cos y-\sin x\sin y}{\cos x\cos y}$$

$$= \frac{\cos x\cos y+\sin x\sin y}{\cancel{\cos x\cos y}} \cdot \frac{\cancel{\cos x\cos y}}{\cos x\cos y-\sin x\sin y}$$

$$= \frac{\cos x\cos y+\sin x\sin y}{\cos x\cos y-\sin x\sin y}$$

$$= \frac{\cos(x-y)}{\cos(x+y)}$$

L.S. = R.S.

QED

b) $\dfrac{\cos 2x+1}{\sin 2x} = \cot x$

Hint: Start working with the more detailed side.

$$\text{L.S.} = \frac{\cos 2x+1}{\sin 2x}$$

$$= \frac{(2\cos^2 x-1)+1}{2\sin x\cos x}$$

$$= \frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x\,\cancel{\cos x}}$$

$$= \frac{\cos x}{\sin x}$$

$$\text{R.S.} = \cot x$$

$$= \frac{\cos x}{\sin x}$$

L.S. = R.S.

QED

c) $\tan 2x - 2\tan 2x \sin^2 x = \sin 2x$

Hint: Can anything be factored out?

$$\text{L.S.} = \tan 2x - 2\tan 2x\sin^2 x$$

$$= \tan 2x(1-2\sin^2 x)$$

$$= \tan 2x\cos 2x$$

$$= \frac{\sin 2x}{\cancel{\cos 2x}} \cdot \frac{\cancel{\cos 2x}}{1}$$

$$= \sin 2x$$

$$\text{R.S.} = \sin 2x$$

L.S. = R.S.

QED