Hmwk: pg 417 # 5bd, 9bc, 10bdf, 11bdfhjl, 16b

Proving Trigonometric Identities: Part 2
More Strategies

1. Some trigonometric identities can be factored as differences of squares as follows

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$
$$\cos^2\theta = 1 - \sin^2\theta = (1-\sin\theta)(1+\sin\theta)$$
$$\sin^2\theta = 1 - \cos^2\theta = (1-\cos\theta)(1+\cos\theta)$$

2. The numerator or denominator can sometimes be turned into a difference of squares factorable expression by multiplying the top and bottom by the conjugate:
   $(a + b)$ ---> conjugate---> $(a - b)$
   $(a - b)$ ---> conjugate ---> $(a + b)$

3. It's always possible to convert all trigonometric components to sines and cosines:

$$\tan\theta = \frac{\sin\theta}{\cos\theta},\ \csc\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

4. The squared trig functions $\sin^2\theta$ and $\cos^2\theta$ can be swapped easily with each other but their linear forms $\sin\theta$ and $\cos\theta$ cannot.

Practice

Prove the following:

a) $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$

L.S. $= \dfrac{\sin^2\theta}{1-\cos\theta}$

$= \dfrac{1-\cos^2\theta}{1-\cos\theta}$

$= \dfrac{\cancel{(1-\cos\theta)}(1+\cos\theta)}{\cancel{(1-\cos\theta)}}$

$= 1+\cos\theta$

R.S. $= 1+\cos\theta$

L.S. = R.S.

QED

b) $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta}=\csc\theta$

$$L.S. = \frac{1+\sec\theta}{\tan\theta+\sin\theta}$$

$$= \frac{1+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\sin\theta}{1}\frac{\cos\theta}{\cos\theta}}$$

$$= \frac{\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}$$

$$= \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}$$

$$= \frac{\cos\theta+1}{\cos\theta} \div \frac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}$$

$$= \frac{\cos\theta+1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta+\sin\theta\cos\theta}$$

$$= \frac{\cancel{(\cos\theta+1)}}{\sin\theta\cancel{(1+\cos\theta)}}$$

$$= \frac{1}{\sin\theta}$$

$$R.S. = \csc\theta$$

$$= \frac{1}{\sin\theta}$$

L.S. = R.S.

QED

c) $\dfrac{\cos\theta}{1-\sin\theta}=\sec\theta+\tan\theta$

$$L.S. = \frac{(\cos\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$R.S. = \sec\theta+\tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

L.S. = R.S.

QED