$1 < x \leq 3$
$(1, 3]$

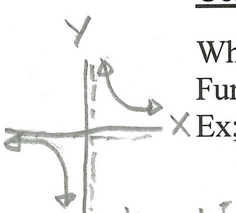
Properties of Functions

Functions can be described based on different criteria:

- domain and range
- x-intercepts and y-intercepts
- continuities and discontinuities
- intervals of increase/decrease
- symmetry (even/odd)
- end behaviours

{x-int (y = 0)
{y-int (x = 0)

Continuous/Discontinuous

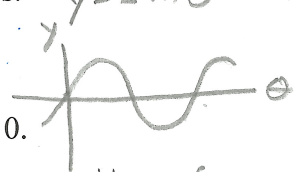


discontinuous
V.A. @ x=0

When a graphed function has no breaks along its domain, it is said to be continuous. Functions that have holes or vertical asymptotes are said to be discontinuous.
Ex; $y = \sin\theta$ is a continuous function.

$y = \frac{1}{x}$ is not a continuous function since there is a break (v. asymptote) at x = 0.

y = sin θ

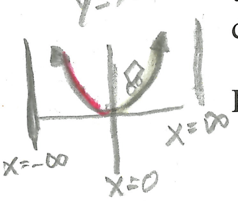


continuous

Intervals of Increase/Decrease

The intervals of increase are regions on the graph where the function is going up from left to right. Conversely, the intervals of decrease are regions where the function is going down.

$y = x^2$

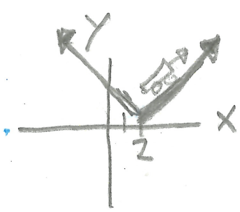


x_1 x_2 x_1 x_2

Ex; y = x^2 The interval of increase is (0, ∞) and the interval of decrease is (-∞, 0).

$y = |x - 2|$ The interval of increase is (2, ∞) and the interval of decrease is (-∞, 2).

x_1 x_2 x_1 x_2



Symmetry(Even/ Odd)

Even Function - is symmetrical about the y-axis; equationally, f(x) = f(-x).
In other words, you get the same graph if you flip it about the y-axis.

Example

Show that f(x) = x^2 is an even function.

$f(-x) = (-x)^2$
$= (-1)^2(x)^2$
$f(-x) = 1x^2$

∴ Since F(x) = F(-x), it has even symmetry.

Summary
even (=) f(x), f(-x)
f(-x), -f(x) (=) odd

Odd Function - has rotational symmetry; f(-x) = -f(x) or -f(-x) = f(x).
In other words, you get the same graph when you flip it about the x-axis then again about the y-axis (or same graph results from rotating it 180° clockwise about the origin).

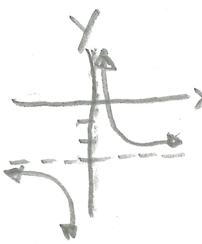
Example

Show that f(x) = $2x^3$ is an odd function.

$f(-x) = 2(-x)^3$
$= 2(-1)^3(x)^3$
$= -2x^3$

$-F(x) = -(2x^3)$
$= -2x^3$

∴ Since f(-x) = -f(x) it has odd symmetry



End Behaviours

End behaviours describe what happens to the function at both extremes of its domain.

Ex; $y=\frac{1}{x}-3$ End behaviour: As x --> ∞, y--> -3^{+}. As x --> -∞, y --> -3^{-}.

goes

Practice

Graph the following functions then complete the table below.

x-int (y=0)

$0=\frac{1}{x-3}+4$

$\frac{-4}{1}=\frac{1}{x-3}$

$-4x+12=1$

$-4x=1-12$

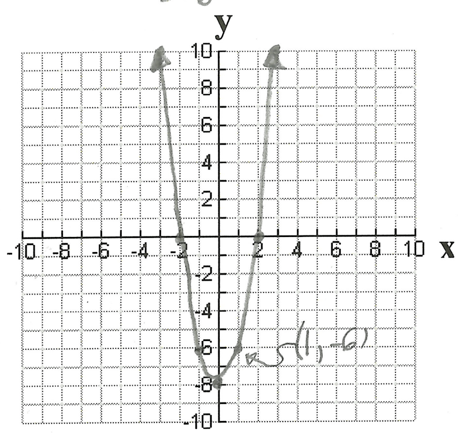
$\frac{-4x}{-4}=\frac{-11}{-4}$

x-int = 2.75

a) $y=2x^2-8$

$=2(1)^2-8$

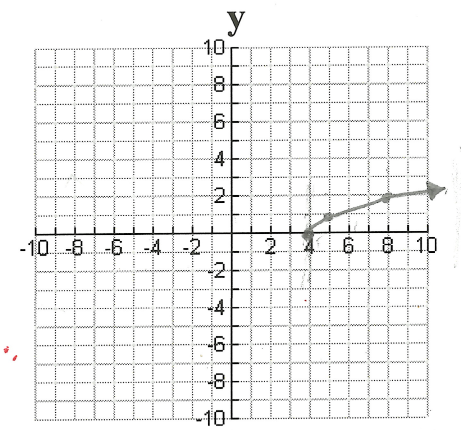
$=-6$



Domain	$\{x \in \mathbb{R}\}$
Range	$\{y \in \mathbb{R} \mid y \geq -8\}$
Intervals of Increase	$(0, \infty)$
Intervals of Decrease	$(-\infty, 0)$
Discontinuities & Asymptotes	None
x-intercepts	± 2
y-intercepts	-8
symmetry	even
end behaviours	

as $x \to -\infty$, $y \to \infty$
as $x \to \infty$, $y \to \infty$

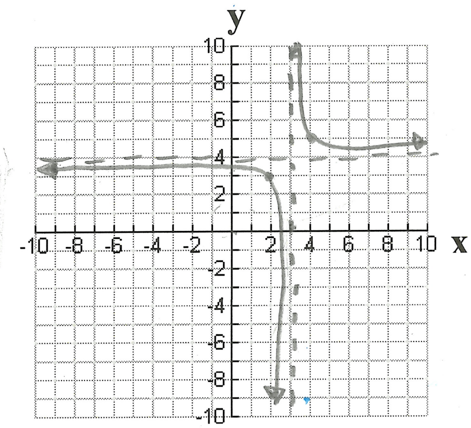
b) $f(x)=\sqrt{x-4}$



Domain	$\{x \in \mathbb{R} \mid x \geq 4\}$
Range	$\{y \in \mathbb{R} \mid y \geq 0\}$
Intervals of Increase	$(4, \infty)$
Intervals of Decrease	None
Discontinuities & Asymptotes	None
x-intercepts	4
y-intercepts	None
symmetry	None
end behaviours	

as $x \to 4$, $y \to 0$
as $x \to \infty$, $y \to \infty$

c) $f(x)=\frac{1}{x-3}+4$



Domain	$\{x \in \mathbb{R} \mid x \neq 3\}$
Range	$\{y \in \mathbb{R} \mid y \neq 4\}$
Intervals of Increase	None
Intervals of Decrease	$(-\infty, 3)$ & $(3, \infty)$
Discontinuities & Asymptotes	V.A. @ x=3
x-intercepts	2.75
y-intercepts	~3.67
symmetry	None
end behaviours	

as $x \to -\infty$, $y \to 4^{-}$
as $x \to \infty$, $y \to 4^{+}$

Homework: pg 23 # 4, 5acde, 6bcd, 7, 8, (10), 12

($\cancel{y=\sin\theta}$) don't do

y-int (x=0)

$y=\frac{1}{0-3}+4$

$y=-\frac{1}{3}+\frac{12}{3}$

$y=\frac{11}{3}$

→ y-int ≈ 3.67