Unit 2 - MHF4U

Review Polynomials (Solutions)

1a) $y = x^3 + 2x^2 + 6x + 2$

odd degree

positive leading coefficient

$\therefore$ as $x \to -\infty$, $y \to -\infty$

as $x \to +\infty$, $y \to +\infty$

b) $y = -x^6 + 2x^3 - 5x + 1$

even degree

negative leading coefficient

$\therefore$ as $x \to -\infty$, $y \to -\infty$

as $x \to +\infty$, $y \to -\infty$

2. See graph on back page.

3.

1st diff.	x	y	1st diff	2nd diff	3rd diff	4th diff
	4	80				
2	6	15	-65			
2	8	0	-15	50		
2	10	-1	-1	14	-36	
2	12	0	1	2	-12	24
2	14	15	15	14	12	24
2	16	80	65	50	36	24
2	18	255	175	110	60	24

The fourth diff. are constant; therefore the relationship is quartic

4. $y = a(x-1)^2(x-4)(x-6)$

sub in $(0, -48)$ ← y-int

$-48 = a(-1)^2(-4)(-6)$

$-48 = 24a$

$\frac{-48}{24} = \frac{24a}{24}$

$a = -2$

$y = -2(x-1)^2(x-4)(x-6)$

5. $2x^2+4x-1 \overline{)\ 8x^4 + 10x^3 - 16x^2 + 4x - 9}$ quotient: $4x^2 - 3x$, R $x - 9$

$8x^4 + 16x^3 - 4x^2$

$-6x^3 - 12x^2 + 4x$

$-6x^3 - 12x^2 + 3x$

$x - 9$

6. $x^4 - 3x^3 + 2x^2 - x + 2 \div x - 5$

$$\begin{array}{r|rrrrr} 5 & 1 & -3 & 2 & -1 & 2 \\ & & 5 & 10 & 60 & 295 \\ \hline & 1 & 2 & 12 & 59 & 297 \end{array}$$

$= x^3 + 2x^2 + 12x + 59, \; R\,297$

7. $\underbrace{x^3 - 6x^2 + 8x - 2}_{F(x)} \div x - 3$

Remainder $= F(3)$

$= (3)^3 - 6(3)^2 + 8(3) - 2$

$= 27 - 54 + 24 - 2$

$= -5$

8. a) $2x^3 + x^2 - 22x + 24$

$f(2) = 0$

$\therefore x-2$ is a factor

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -22 & 24 \\ & & 4 & 10 & -24 \\ \hline & 2 & 5 & -12 & 0 \end{array}$$

$= (x-2)(2x^2+5x-12)$

$= (x-2)(2x-3)(x+4)$

Decomp.

$2x^2 + 5x - 12$

$= 2x^2 + 8x - 3x - 12$

$= 2x(x+4) - 3(x+4)$

$= (2x-3)(x+4)$

$\left.\begin{array}{l} P(-24) \\ S(5) \end{array}\right\} 8, -3$

diff. of cubes → b) $8x^3 - 125$

$a = 2x \quad b = 5$

$= (a-b)(a^2+ab+b^2)$

$= (2x-5)[(2x)^2 + (2x)(5) + (5)^2]$

$= (2x-5)(4x^2+10x+25)$

use grouping

c) $2x^3 + 3x^2 - 8x - 12$

$= x^2(2x+3) - 4(2x+3)$

$= (x^2-4)(2x+3)$

$= (x-2)(x+2)(2x+3)$

9. $12x+18 < 2x^4+12x^3+16x^2$

$-2x^4-12x^3-16x^2+12x+18<0$

$-2\underbrace{(x^4+6x^3+8x^2-6x-9)}_{f(x)}<0$

$f(1)=0$

$\therefore (x-1)$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & 6 & 8 & -6 & -9 \\ & & 1 & 7 & 15 & 9 \\ \hline & 1 & 7 & 15 & 9 & 0 \end{array}$$

$-2(x-1)(\underbrace{x^3+7x^2+15x+9}_{g(x)})<0$

$g(-3)=0$

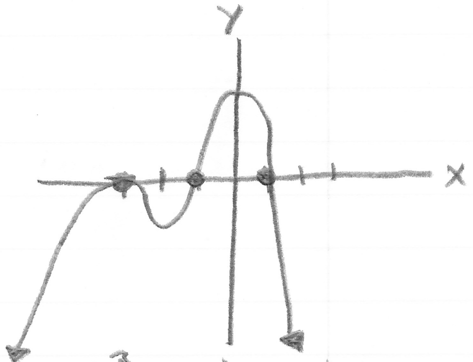
$\therefore$ $x+3$ is a factor

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 15 & 9 \\ & & -3 & -12 & -9 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$-2(x-1)(x+3)(x^2+4x+3)<0$

$-2(x-1)(x+3)(x+3)(x+1)<0$

$-2(x-1)(x+3)^2(x+1)<0$



or

	-3	-1	1	
-2	$-$	$-$	$-$	$-$
$(x-1)$	$-$	$-$	$-$	$+$
$(x+3)^2$	$+$	$+$	$+$	$+$
$(x+1)$	$-$	$-$	$+$	$+$
Product	$-$	$-$	$+$	$-$

$x<-3$ or $-3<x<-1$ or $x>1$

10 a) $AROC = \dfrac{f(2)-f(0)}{2-0}$

$= \dfrac{-3-(3)}{2}$

$=-3$

b) $IROC \cong \dfrac{f(-0.99)-f(-1)}{0.01}$

$\cong \dfrac{0.079401-0}{0.01}$

$\cong 7.94$

Verify

$y' = 3x^2-6x-1$

at $x=-1$

$y' = 3(-1)^2-6(-1)-1$

$= 3+6-1$

$=8$

Close to $\boxed{7.94}$ so we are good

:)

Review Questions

1. State the end behaviors for the following

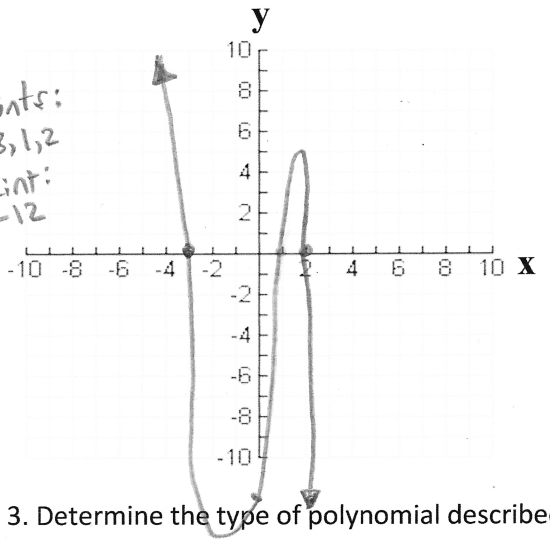
   a) $y = x^3 + 2x^2 + 6x + 2$ b) $y = -x^6 + 2x^3 - 5x + 1$

2. Sketch a graph for the following; include all x-intercepts and the y-intercept.

   a) $y = -2(x+3)(x-2)(x-1)$ b) $f(x) = -2x^4 + 12x^3 - 16x^2 - 12x + 18$

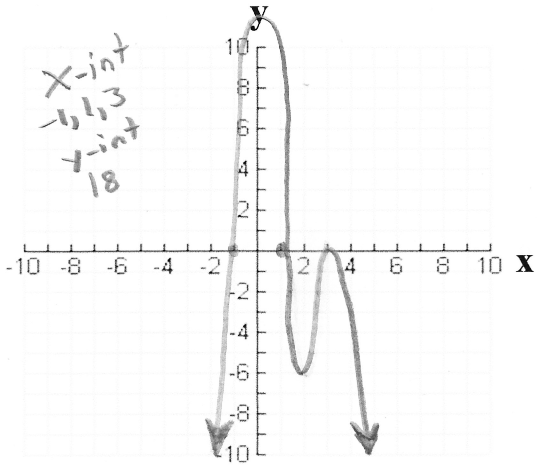
x-ints: -3, 1, 2

y-int: -12



X-int -1, 1, 3

y-int 18



$f(x) = -2x^4 + 12x^3 - 16x^2 - 12x +$

$= -2\underbrace{(x^4 - 6x + 8x^2 + 6x - 9)}_{g(x)}$

$g(1) = 0$

$\therefore x-1$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & 8 & 6 & -9 \\ & & 1 & -5 & 3 & 9 \\ \hline & 1 & -5 & 3 & 9 & 0 \end{array}$$

$= -2(x-1)\underbrace{(x^3 - 5x^2 + 3x + 9)}_{h(x)}$

$h(3) = 0$

$\therefore x-3$ is a factor

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 3 & 9 \\ & & 3 & -6 & -9 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$= -2(x-1)(x-3)(x^2 - 2x - 3)$

$= -2(x-1)(x-3)(x-3)(x+$

$= -2(x-1)(x-3)^2(x+$

3. Determine the type of polynomial described in the following table of values using finite differences.

X	Y
4	80
6	15
8	0
10	-1
12	0
14	15
16	80
18	255

4. Find the equation of the quartic function that has a turning point at the x-intercept 1, two other x-intercepts at 4 and 6 and a y-intercept of -48.

5. Use long division to divide the following: $(8x^4 + 10x^3 - 16x^2 + 4x - 9) \div (2x^2 + 4x - 1)$.

6. Use synthetic division to evaluate the following: $(x^4 - 3x^3 + 2x^2 - x + 2) \div (x - 5)$.

7. Determine the remainder: $(x^3 - 6x^2 + 8x - 2) \div (x - 3)$.

8. Factor.

   a) $2x^3 + x^2 - 22x + 24$ b) $8x^3 - 125$ c) $2x^3 + 3x^2 - 8x - 12$

9. Solve the following inequality: $12x + 18 < 2x^4 + 12x^3 + 16x^2$

10. For the function $f(x) = x^3 - 3x^2 - x + 3$...

a) Evaluate the rate of change over the interval $0 \le x \le 2$.

b) Evaluate the rate of change at $x = -1$ then verify your answer using the derivative.