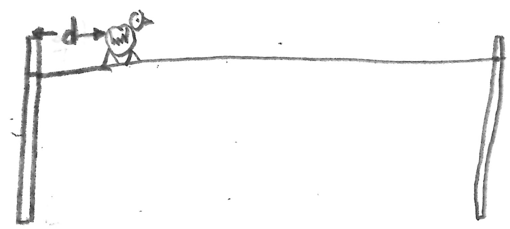
Polynomial Functions and Rates of Change: Extra Practice

A bird is walking along a wire. Its position (displacement) from the left most point on the wire is given by the equation:

$$d(t) = t^3 - 12t^2 + 36t + 5$$



where

- d is measured in metres
- t is the time measured in seconds

a) How fast is the bird traveling (and in what direction) when t = 1 second?

$$IROC \cong \frac{d(1.01) - d(1)}{0.01}$$

$$\cong \frac{30.149101 - 30}{0.01}$$

$\cong 14.9 m/s$ [right]

b) Verify your answer in part a) using the derivative.

$d(t) = 1t^3 - 12t^2 + 36t + 5$

$d'(t) = 3t^2 - 24t + 36$

$d'(1) = 3(1)^2 - 24(1) + 36$

$d'(1) = 15 m/s$

c) How fast is the bird traveling (and in what direction) from 3 seconds to 5 seconds?

$$AROC = \frac{d(5) - d(3)}{5 - 3}$$

$$= \frac{10 - 32}{2}$$

$= -11 m/s$

$\boxed{= 11 m/s \text{ [left]}}$

d) Create a detailed graph of this function and use it to determine the total distance traveled from 0 seconds to 10 seconds; use the derivative to determine the coordinates of the local max and min.

end points

At $t = 0$, $d(0) = 5$

At $t = 10$, $d(10) = 165$

stat. points

$d'(t) = 3t^2 - 24t + 36$

set $d'(t) = 0$

$0 = 3t^2 - 24t + 36$

$0 = 3(t^2 - 8t + 12)$

$0 = 3(t-2)(t-6)$

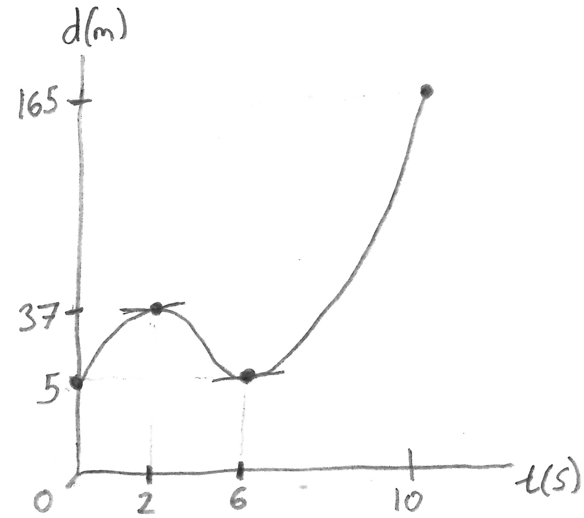
$t = 2$ or $t = 6$

At $t = 2$, $d(2) = 37$

At $t = 6$, $d(6) = 5$

distance $= 32 + 32 + 160$

$= 224 m$



Optimization (Extension)

1. Consider the function $f(x) = 3x^4 - 16x^3 + 18x^2$. Determine the location of all local max/min.

$f'(x) = 12x^3 - 48x^2 + 36x$

set $f'(x) = 0$

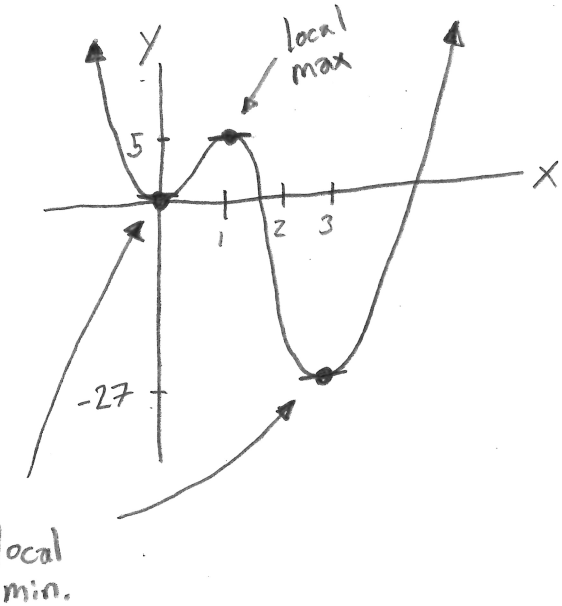
$0 = 12x^3 - 48x^2 + 36x$

$0 = 12x(x^2 - 4x + 3)$

$0 = 12x(x-1)(x-3)$

$x = 0, 1, 3$

stat. points
$$\begin{cases} \text{At } x=0, & f(0)=0 \\ \text{At } x=1, & f(1)=5 \\ \text{At } x=3, & f(3)=-27 \end{cases}$$



2. Global maximums and minimums can occur at any of three places: at turning points, at the end points, or on the edge of vertical asymptotes. Polynomial functions do not have asymptotes. Without graphing, determine the global minimum and maximum values for the function $f(x) = x^4 - 4x^3 - 8x^2$.

1. Edge of Vertical Asymptotes

   Not applicable for polynomial functions

2. End Points

   As $x \to -\infty, y \to \infty$

   As $x \to \infty, y \to \infty$

3. Stat. Points

   $f'(x) = 4x^3 - 12x^2 - 16x$

   set $f'(x) = 0$

   $0 = 4x^3 - 12x^2 - 16x$

   $0 = 4x(x^2 - 3x - 4)$

   $0 = 4x(x+1)(x-4)$

   $x = 0, -1, 4$

At $x = 0$, $f(0) = 0$

At $x = -1$, $f(-1) = -3$

At $x = 4$, $f(4) = -128$

$\therefore$ Global Max

As $x \to \pm\infty, y \to \infty$

Global Min

At $x = 4$, $f(4) = -128$