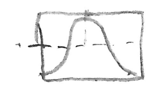
Modelling with Trigonometric Functions

(Sinusoidal)

Recall:

For sinusoidal functions....

- k is obtained from the period or frequency: $k = \frac{2\pi}{T}$ or $k = 2\pi f$
  - * It is rare to ever use a negative value for k when modelling real-life scenarios.
- d is the phase shift or horizontal shift to the left side of the box enclosing one cycle of the sinusoidal function.

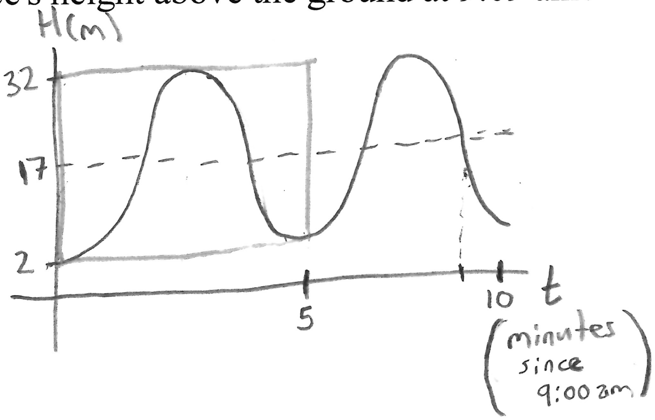


- |a| is the amplitude of the function.
  - * Sometimes we use a negative value for 'a' in situations when it is convenient to do so; ie… to eliminate any phase shift.
- c is the location of the axis of equilibrium.

Example 1

Renee DesCartes boards the Pythagorean Ferris Wheel at 9:00 am. The base of the wheel is 2 m above ground. The diameter of the wheel is 30 m. If it takes 5 minutes for the wheel to complete one full revolution, what is Renee's height above the ground at 9:09 am?

(1)



(3) $H = a\cos[k(t-d)] + c$

$H = -15\cos\left[\frac{2\pi}{5}(t)\right] + 17$

$H = -15\cos\left(\frac{2\pi t}{5}\right) + 17$

(4) set $t = 9$

$H = -15\cos\left(\frac{2\pi(9)}{5}\right) + 17$

$H \doteq 12.4\text{m}$

(2) $k = \frac{2\pi}{T}$

$= \frac{2\pi}{5}$

$d = 0$

$a = -15$

$c = 17$

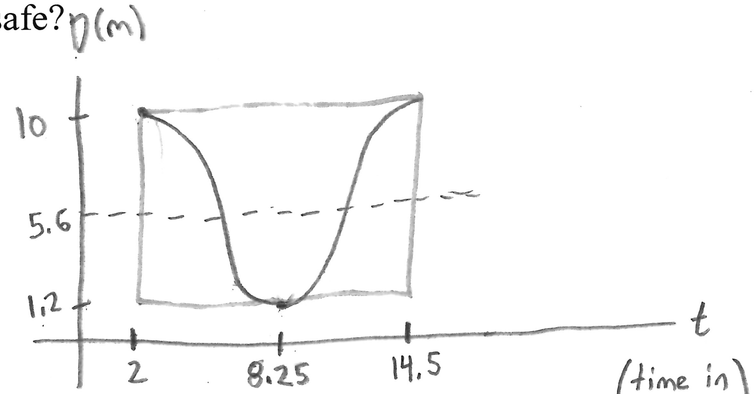
Notice to solve the problem in example 1, we did the following:
1. Create a sketch of a sinusoidal graph to model the scenario.
2. Put a box around one cycle of a sine or cosine period then determine the parameters k, d, a, and c.
3. Create the sinusoidal function using your values of k, d, a, and c.
4. Plug in a value for one of the two variables and solve.

Example 2

The tides at Cape Capstan change the depth of the water in the harbour. On one day in October, the tides have a high point of approximately 10 m at 2 pm and a low point of 1.2 m at 8:15 pm. A particular sailboat has a draft of 2m; this means it can only move in water that is at least 2 m deep.

a) The captain of the sailboat plans to exit the harbour at 6:30 pm. Is this safe?

(1)



(2) $K = \frac{2\pi}{T} = \frac{2\pi}{12.5} = \frac{4\pi}{25}$

$d = 2$

$a = 4.4$

$c = 5.6$

(3) $D = a\cos[k(t-d)] + c$

$D = 4.4\cos\left[\frac{4\pi}{25}\frac{(t-2)}{1}\right] + 5.6$

$D = 4.4\cos\left[\frac{4\pi(t-2)}{25}\right] + 5.6$

Set $t = 6.5$

(4) $D = 4.4\cos\left[\frac{4\pi(6.5-2)}{25}\right] + 5.6$

$D \doteq 2.8m$

∴ Since the depth is larger than 2m, it is safe.

b) When is it safe to return?

Set $D = 2m$

$4.4\cos\left[\frac{4\pi(t-2)}{25}\right] + 5.6 = 2$

$\frac{4.4\cos\left[\frac{4\pi(t-2)}{25}\right]}{4.4} = \frac{2-5.6}{4.4}$

$\cos\left(\frac{4\pi(t-2)}{25}\right) = \frac{-3.6}{4.4}$

$\frac{4\pi(t-2)}{25} = \cos^{-1}\left(-\frac{3.6}{4.4}\right)$

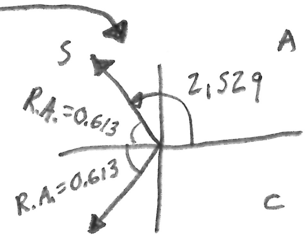
$\frac{4\pi(t-2)}{25} = \frac{2.529}{1}$

$\frac{4\pi(t-2)}{4\pi} = \frac{25(2.529)}{4\pi}$

$t - 2 = 5.03$

$t = 7.03$

(7:02 pm)



$\frac{4\pi(t-2)}{25} = \frac{3.754}{1}$

$t = 9.469$

(9:28 pm)

∴ The captain can return before 7:02 pm or after 9:28 pm.

Tides