Logarithms and Exponentials: Practice

1. Solve for x **without** using a calculator.

a) $x = \log_3 81$

b) $x = \log_6 6\sqrt{6}$

c) $x = \log(-100)$

d) $x = \log_M M^2$

e) $4 = \log_2 x$

f) $\frac{1}{16} = 2^x$

g) $x = R^{\log_R Q}$

h) $x = \log_5 5^{71}$

i) $x = \log 100000$

2. Determine the inverse of each function.

a) $y = 2^{x-3}$

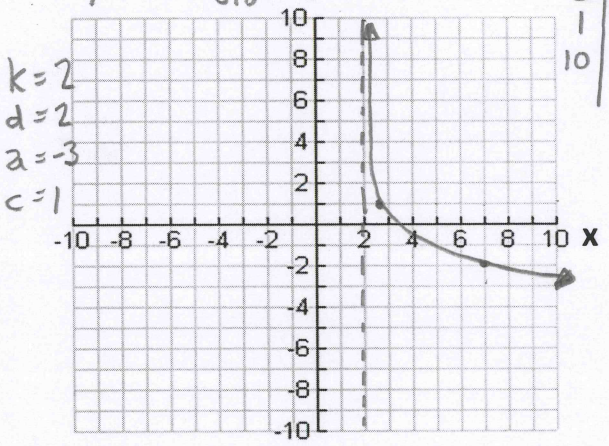
b) $y = \log_4(x-1) + 5$

3. Graph the following and state the domain and range.

a) $y = -3\log_{10}(2x-4) + 1$

$y = -3\log_{10}[2(x-2)] + 1$

x	$y = \log_{10} x$
0	V.A
1	0
10	1

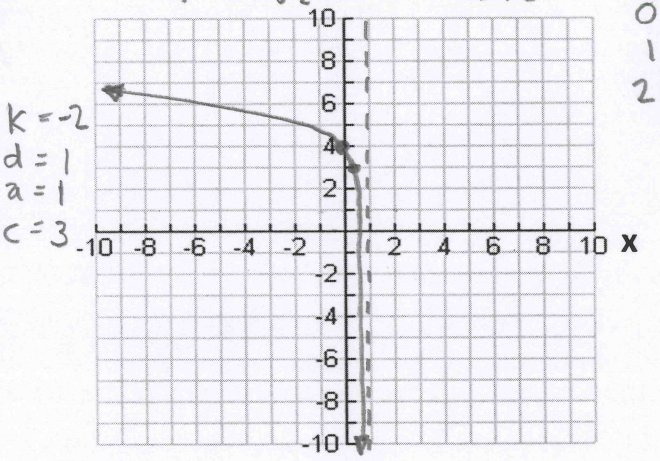


Domain: $\{x \in \mathbb{R} \mid x > 2\}$ Range: $\{y \in \mathbb{R}\}$

b) $y = \log_2(-2x+2) + 3$

$y = \log_2[-2(x-1)] + 3$

x	$y = \log_2 x$
0	V.A.
1	0
2	1



Domain: $\{x \in \mathbb{R} \mid x < 1\}$ Range: $\{y \in \mathbb{R}\}$

4. Solve each exponential equation for x.

a) $2(3)^x = 50$

b) $4^{x+1} = 8^{2x-5}$

c) $2^{x+3} + 2^x = 36$

d) $2^{x+2} = 7^{2x-3}$

e) $2^{x+1}5^{x-2} = 12$

f) $3(3^{2x}) + 26(3^x) = 9$

5. Write the following as sums and differences of logarithmic expressions where each term is in the form $n\log_a x$

a) $\log_2\left(\frac{x^3}{y}\right)^4$

b) $\log_3 \sqrt[5]{(xy^2)^4}$

6. Solve each logarithmic equation for x.

a) $\log_x 9 = 2$

b) $\log_5(3x - 7) = \log_5 8$

c) $\log_3 3x + \log_3 6 = 2$

d) $\log_3(x + 6) - \log_3(x + 5) = 1$

e) $\log_2(x + 2) + \log_2(x - 5) = 3$

f) $\log_4 x - \log_5 x = 2$

7. How much more intense is the sound emitted from a lawnmower (107 dB) in comparison to normal conversation (65dB)?

$$Loudness(in\ dB) = 10\log\left(\frac{I}{I_O}\right) \text{ where } I_O = 10^{-12} W/m^2$$

8. A student figures that their grade on the final exam, G, in percent, is a function of the total amount of time studying, t in hours, prior to the assessment as follows:

$$G(t) = 40\log(t + 1) + 50$$

a) What grade should this student expect to get on the final exam if they study for 2 hours?

b) At what rate will the grade increase when the amount of studying increases from 2 hours to 5 hours?

c) At what rate is the grade increasing when this student studies for only 1 hour?

Answers:

1. a) 4 b) 3/2 c) No Soln d) 2 e) 16 f) -4 g) Q h) 71 i) 5

2. a) $y = \log_2 x + 3$ b) $y = 4^{x-5} + 1$

3. See graph online.

4. a) 2.93 b) 4.25 c) 2 d) 2.26 e) 2.18 f) -1

5. a) $12\log_2 x - 4\log_2 y$ b) $\frac{4}{5}\log_3 x + \frac{8}{5}\log_3 y$

6. a) 6 b) 5 c) ½ d) -4.5 e) 6 f) 483945637.8

7. $I_L = 10^{-1.3}$ W/m^2, $I_C = 10^{-5.5}$ W/m^2, Lawnmower sound 15849 times more intense.

8. a) 69% b) 4.01%/hour c) 8.66%/hour

Logarithms and Exponentials: Practice

1a) $x = \log_3 81$

$x = 4$

b) $x = \log_6 6\sqrt{6}$

$= \log_6 6^1 \cdot 6^{\frac{1}{2}}$

$= \log_6 6^{3/2}$

$= 3/2$

c) $x = \log(-100)$

No Soln

d) $x = \log_M M^2$

$= 2\log_M M$

$= 2(1)$

$= 2$

e) $4 = \log_2 x$

$x = 2^4$

$x = 16$

f) $\frac{1}{16} = 2^x$

$2^{-4} = 2^x$

$x = -4$

g) $x = R^{\log_R Q}$

$= Q$

h) $x = \log_5 5^{71}$

$= 71$

i) $x = \log 100000$

$= 5$

2a) $y = 2^{x-3}$

$x = 2^{y-3}$

$y - 3 = \log_2 x$

$y = \log_2 x + 3$

inverse

b) $y = \log_4 (x-1) + 5$

$x = \log_4 (y-1) + 5$

$x - 5 = \log_4 (y-1)$

$y - 1 = 4^{x-5}$

$y = 4^{x-5} + 1$

inverse

3. See graph

4a) $\frac{2(3^x)}{2} = \frac{50}{2}$

$3^x = 25$

$x = \log_3 25$

$x = \frac{\log 25}{\log 3}$

$x \approx 2.93$

b) $4^{x+1} = 8^{2x-5}$

$(2^2)^{x+1} = (2^3)^{2x-5}$

$2^{2x+2} = 2^{6x-15}$

$2x + 2 = 6x - 15$

$2x - 6x = -15 - 2$

$\frac{-4x}{-4} = \frac{-17}{-4}$

$x = 4.25$

c) $2^{x+3} + 2^x = 36$

$2^x 2^3 + 2^x = 36$

$2^x(2^3+1) = 36$

$\frac{2^x(9)}{9} = \frac{36}{9}$

$2^x = 4$

$2^x = 2^x$

$x = 2$

d) $2^{x+2} = 7^{2x-3}$

$\log 2^{x+2} = \log 7^{2x-3}$

$(x+2)\log 2 = (2x-3)\log 7$

$x\log 2 + 2\log 2 = 2x\log 7 - 3\log 7$

$x\log 2 - 2x\log 7 = -3\log 7 - 2\log 2$

$\frac{x(\log 2 - 2\log 7)}{\log 2 - 2\log 7} = \frac{-3\log 7 - 2\log 2}{\log 2 - 2\log 7}$

$x \approx 2.26$

e) $2^{x+1} 5^{x-2} = 12$

$\log(2^{x+1} 5^{x-2}) = \log 12$

$\log 2^{x+1} + \log 5^{x-2} = \log 12$

$(x+1)\log 2 + (x-2)\log 5 = \log 12$

$x\log 2 + \log 2 + x\log 5 - 2\log 5 = \log 12$

$x\log 2 + x\log 5 = \log 12 - \log 2 + 2\log 5$

$\frac{x(\log 2 + \log 5)}{\log 2 + \log 5} = \frac{\log 12 - \log 2 + 2\log 5}{\log 2 + \log 5}$

$x \approx 2.18$

f) $3(3^{2x}) + 26(3^x) = 9$

$3(3^x)^2 + 26(3^x) - 9 = 0$

Let $n = 3^x$

$3n^2 + 26n - 9 = 0$

$3n^2 - n + 27n - 9 = 0$ $\quad P(-27)\ S(26)\ \} -1, 27$

$n(3n-1) + 9(3n-1) = 0$

$(n+9)(3n-1) = 0$

$n = -9$ or $n = \frac{1}{3}$

$3^x = -9$ $\quad 3^x = 1/3$

No Sol^n or $3^x = 3^{-1}$

$x = -1$

5. a) $\log_2 \left(\frac{x^3}{y}\right)^4$

$= \log_2 \left(\frac{x^{12}}{y^4}\right)$

$= \log_2 x^{12} - \log_2 y^4$

$= 12\log_2 x - 4\log_2 y$

b) $\log_3 \sqrt[5]{(xy^2)^4}$

$= \log_3 (xy^2)^{\frac{4}{5}}$

$= \frac{4}{5}\log_3 (xy^2)$

$= \frac{4}{5}[\log_3 x + \log_3 y^2]$

$= \frac{4}{5}[\log_3 x + 2\log_3 y]$

$= \frac{4}{5}\log_3 x + \frac{8}{5}\log_3 y$

6. a) $\log_x 9 = 2$

$\sqrt{x^2} = \pm\sqrt{9}$

$x = 3$ or ~~$x = -3$~~ (inadmissable)

b) "unlog"

$\log_5(3x-7) = \log_5 8$

$3x - 7 = 8$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

c) $\log_3 3x + \log_3 6 = 2$

$\log_3[(3x)(6)] = 2$

$18x = 3^2$

$\frac{18x}{18} = \frac{9}{18}$

$x = \frac{1}{2}$

d) $\log_3(x+6) - \log_3(x+5) = 1$

$\log_3\left(\frac{x+6}{x+5}\right) = 1$

$\frac{x+6}{x+5} = 3^1$

$\frac{x+6}{x+5} = \frac{3}{1}$

$3(x+5) = x+6$

$3x + 15 = x + 6$

$3x - x = 6 - 15$

$\frac{2x}{2} = \frac{-9}{2}$

$x = -4.5$

e) $\log_2(x+2) + \log_2(x-5) = 3$

$\log_2[(x+2)(x-5)] = 3$

$(x+2)(x-5) = 2^3$

$x^2 - 3x - 10 = 8$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x = 6$ or ~~$x = -3$~~ (inadmissable)

f) $\log_4 x - \log_5 x = 2$

$\frac{\log 5 \log x}{\log 5 \log 4} - \frac{\log 4 \log x}{\log 4 \log 5} = 2$

$\frac{\log x \log 5 - \log x \log 4}{\log 5 \log 4} = \frac{2}{1}$

$\frac{\log x(\log 5 - \log 4)}{\log 5 - \log 4} = \frac{2 \log 5 \log 4}{\log 5 - \log 4}$

$\log x = \frac{2 \log 5 \log 4}{\log 5 - \log 4}$

$x = 10^{\frac{2 \log 5 \log 4}{\log 5 - \log 4}}$

$x \approx 483945637.8$

7. Lawnmower

$L_L = 10\log\left(\frac{I_L}{I_0}\right)$

$\frac{107}{10} = \frac{10\log\left(\frac{I_L}{10^{-12}}\right)}{10}$

$10.7 = \log\left(\frac{I_L}{10^{-12}}\right)$

$\frac{I_L}{10^{-12}} = 10^{10.7}$

$I_L = 10^{-12} \cdot 10^{10.7}$

$I_L = 10^{-1.3}\ W/m^2$

Conversation

$L_c = 10\log\left(\frac{I_c}{I_0}\right)$

$\frac{65}{10} = \frac{10\log\left(\frac{I_c}{10^{-12}}\right)}{10}$

$6.5 = \log\left(\frac{I_c}{10^{-12}}\right)$

$\frac{I_c}{10^{-12}} = 10^{6.5}$

$I_c = 10^{-12} \cdot 10^{6.5}$

$I_c = 10^{-5.5}$

$\text{Factor} = \frac{I_L}{I_c}$

$= \frac{10^{-1.3}}{10^{-5.5}}$

$= 10^{4.2}$

≈ 15849

∴ The sound of the lawnmower is approx. 15849 times more intense than normal conversation.

8. a) set $t = 2$

$G = 40\log(2+1) + 50$

$= 40\log 3 + 50$

$\approx 69\%$

b) $AROC = \frac{G(5) - G(2)}{5 - 2}$

$= \frac{81.12605002 - 69.08485019}{3}$

$\approx 4.01\%/\text{hour}$

c) $IROC = \frac{G(1.01) - G(1)}{0.01}$

$\approx \frac{62.1278423 - 62.04119983}{0.01}$

$\approx 8.66\%/\text{hour}$