Hmwk: pg 475 #1-4ace, 5, 6ace, 7, 8, 9-11ace, 16, 17

Logarithms Laws

Recall:

Product Law: $a^x a^y = a^{x+y}$

Quotient Law: $\frac{a^x}{a^y} = a^{x-y}$

Power Law: $(a^x)^y = a^{xy}$

Activity:

Prove the following **logarithm laws** then provide an example of its application:

Product Law

Let $m = a^x$ and $n = a^y$

$mn = a^x a^y$

$mn = a^{x+y}$

$\log_a mn = \log_a a^{x+y}$

① $\log_a mn = x + y$

but...

$m = a^x$ and $n = a^y$

② $x = \log_a m$ ③ $y = \log_a n$

sub ② and ③ into ①

$\log_a mn = \log_a m + \log_a n$

Ex $\log_2 (4 \cdot 16)$

$= \log_2 4 + \log_2 16$

$= 2 + 4$

$= 6$

Quotient Law

Let $m = a^x$ and $n = a^y$

$\frac{m}{n} = \frac{a^x}{a^y}$

$\frac{m}{n} = a^{x-y}$

$\log_a \left(\frac{m}{n}\right) = \log_a a^{x-y}$

① $\log_a \left(\frac{m}{n}\right) = x - y$

but...

$m = a^x$ and $n = a^y$

② $x = \log_a m$ ③ $y = \log_a n$

$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

Ex $\log_2 \left(\frac{32}{4}\right)$

$= \log_2 32 - \log_2 4$

$= 5 - 2$

$= 3$

Power Law

Let $m = a^x$

$m^n = (a^x)^n$

$m^n = a^{nx}$

$\log_a m^n = \log_a a^{nx}$

① $\log_a m^n = nx$

but $m = a^x$...

So... ② $x = \log_a m$

$\log_a m^n = n \log_a m$

Ex $\log_2 4^3$

$= 3 \log_2 4$

$= 3(2)$

$= 6$

Example 1

Simplify each logarithmic expression.

a) $log_2 5 + log_2 6.4$

$= \log_2(5 \cdot 6.4)$

$= \log_2(32)$

$= 5$

b) $log_3 54 - log_3 6$

$= \log_3\left(\frac{54}{6}\right)$

$= \log_3 9$

$= 2$

c) $log_5 25^6$

$= 6 \log_5 25$

$= 6(2)$

$= 12$

Example 2

Determine two different transformations that could be applied to the parent function $y = log\, x$ to create a graph for the function $y = log\, 100x$.

Transformation 1

$y = \log 100x$

$\uparrow$

$k = 100$

→ horizontal compression by a factor of $\frac{1}{100}$

Transformation 2

$y = \log 100x$

$y = \log 100 + \log x$

$y = 2 + \log x$

$y = \log x + 2$

$\uparrow$

$c = 2$

→ vertical shift up 2 units

Example 3

Write the following as sums and differences of logarithmic expressions where each term is in the form $n\, log_a\, x$.

a) $log_4(xy^3)$

$= \log_4 x + \log_4 y^3$

$= \log_4 x + 3\log_4 y$

b) $log_2 \sqrt{\frac{x^3y}{z^2}}$

$= \log_2\left(\frac{x^3y}{z^2}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\log_2\left(\frac{x^3y}{z^2}\right)$

$= \frac{1}{2}\left(\log_2 x^3y - \log_2 z^2\right)$

$= \frac{1}{2}\left(\log_2 x^3 + \log_2 y - \log_2 z^2\right)$

$= \frac{1}{2}\left(3\log_2 x + \log_2 y - 2\log_2 z\right)$

$= \frac{3}{2}\log_2 x + \frac{1}{2}\log_2 y - \log_2 z$