Homework: read pg 450, do pg 451 #1ab, 2ab, 4-11

Introduction to Logarithms

A logarithmic function is the inverse of an exponential; they are a useful function for solving exponents.

The parent logarithmic function is written as:

$$y = \log_a \boxed{x}$$

argument

base

This function is read, "y equals the log of x with base a". When there is no number written for 'a', then the base is ten by default.

The expression $\log_a x$ determines what exponent needs to be placed on 'a' so that it equals x.

Example 1

Evaluate the following.

a) $\log_2 8$

$2^? = 8$

$= 3$

b) $\log_4 16$

$4^? = 16$

$= 2$

c) $\log 1000$

means $\log_{10} 1000$

$= 3$

d) $\log(-100)$

not -2

$10^? = -100$

$= \text{No sol}^{\text{n}}$ (Error)

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

The above expressions can be evaluated using a technique referred to as "splitting". This technique works as follows.

(change the base)

$$\boxed{\log_a x = \frac{\log x}{\log a}}$$

Example 2

Use splitting to evaluate the following.

a) $\log_2 8$

$= \frac{\log 8}{\log 2}$

$= 3$

b) $\log_4 16$

$= \frac{\log 16}{\log 4}$

$= 2$

c) $\log 1000$

$= \frac{\log 1000}{\log 10}$

$= 3$

d) $\log(-100)$

$= \frac{\log(-100)}{\log 10}$

$= \text{Error}$ (No sol^{n})

Switching between logarithmic and exponential form

Ex
a) $y = 8^x$
$x = \log_8 y$
b) $y = \log_x 5$
$x^y = 5$

A basic exponential function and its inverse can be written as follows:

Exponential Function		Inverse	
$y = a^x$	-------->	$x = a^y$	(Exponential Form)
	-------->	$y = \log_a x$	(Logarithmic Form)

equivalent

Example 3

$\boxed{x = a^y \longleftrightarrow y = \log_a x}$

Write the inverse of each exponential function in exponential form and logarithmic form.

a) $y = 3^x$
$x = 3^y$
$y = \log_3 x$

b) $y = 5^{3x}$
$x = 5^{3y}$
$\frac{3y}{3} = \frac{\log_5 x}{3}$
$y = \frac{\log_5 x}{3}$

c) $y = 4^{x-2}$
$x = 4^{y-2}$
$y - 2 = \log_4 x$
$y = \log_4 x + 2$

Example 4

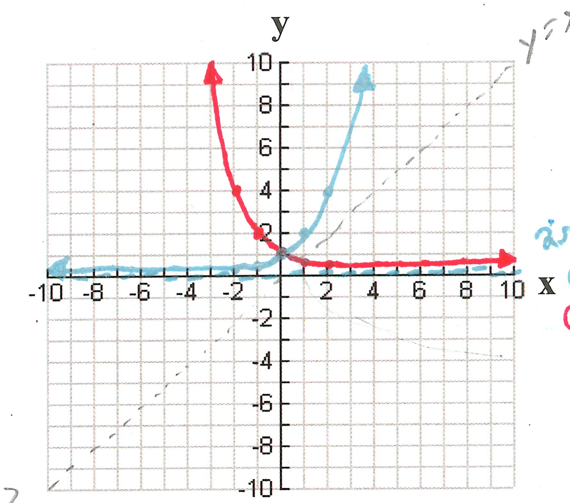
Graph the following sets of functions.

a) Exponential Functions

$y = (2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$

x	$y = 2^x$	$y = \left(\frac{1}{2}\right)^x$
-2	1/4	4
-1	1/2	2
0	1	1
1	2	1/2
2	4	1/4

$y = \left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 4$



asymptote @ y=0
@ y=0

Domain: $\{x \in \mathbb{R}\}$ Range: $\{y \in \mathbb{R} / y > 0\}$

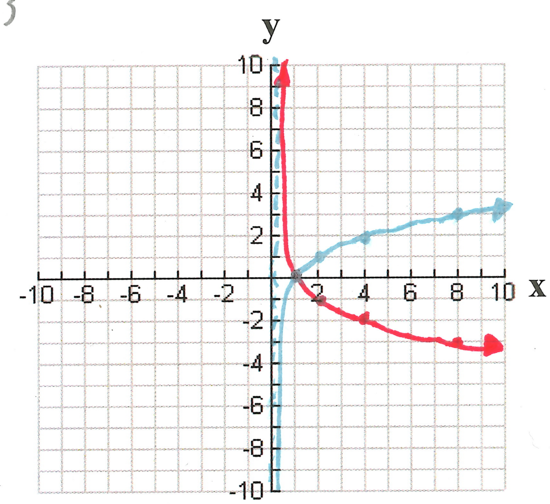
$y = 2^x$
$x = 2^y$
$\boxed{y = \log_2 x}$

b) The Inverse Functions of a)

$\left(\frac{1}{2}\right)^{?} = 2$ -1
$\left(\frac{1}{2}\right)^{?} = 4$ -2

x	$y = \log_2 x$	$y = \log_{\frac{1}{2}} x$
1	0	0
2	1	-1
4	2	-2
8	3	-3

$\log_2 1 = 0$
$\log_2 2 = 1$



V. asymptote @ x=0

Domain: $\{x \in \mathbb{R} / x > 0\}$ Range: $\{y \in \mathbb{R}\}$