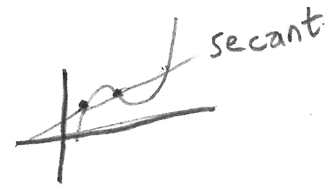
Homework: pg 85 #1a(second table only),3, 4ac, 5, 6, (7), 8, 9, 10(use one method), 14

Instantaneous Rate of Change



Recall:

Secant - a line that passes through two points on the graph.

Average Rate of Change - is determined by calculating the slope of the secant and represents the rate of change over a non-zero interval.

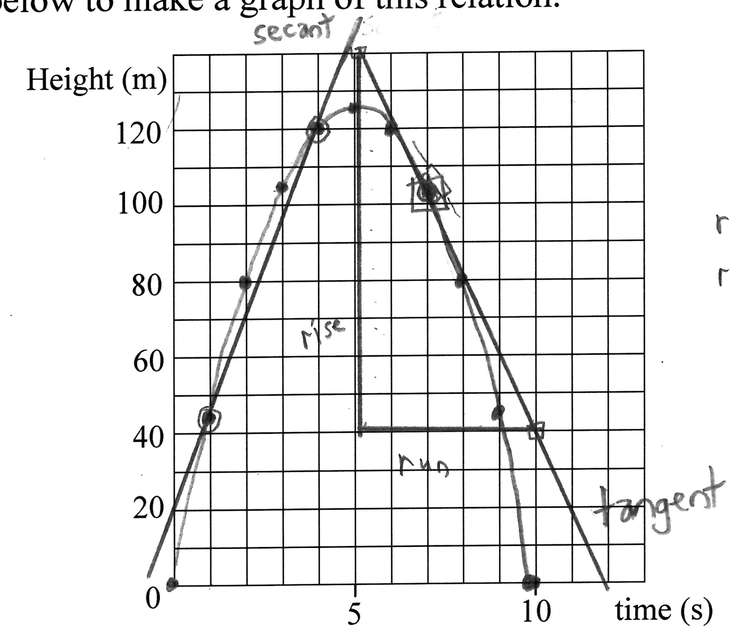
Investigation

Suppose a flare is launched in the air and its height(m) is represented by the following equation where t is time in seconds:

$$H(t) = -5t^2 + 50t$$

Use the table of values below to make a graph of this relation.

time (s)	Height (m)
0	0
1	45
2	80
3	105
4	120
5	125
6	120
7	105
8	80
9	45
10	0



rise = -100m
run = 5s

Draw a secant to represent the interval $1 \leq t \leq 4$ then determine the slope of this line to find the average rate of change for this interval.

$$AROC = \frac{\Delta H}{\Delta t} = \frac{H(4) - H(1)}{4-1} = \frac{120-45}{3} = \frac{75}{3} = 25 m/s$$

The "instantaneous rate of change" is a rate of change at one specific value of the independent variable.

A tangent is a line that can be drawn through any one point on the curve. The direction of the tangent follows the trend of the curve at that specific point. The instantaneous rate of change at that point is the slope of the tangent.

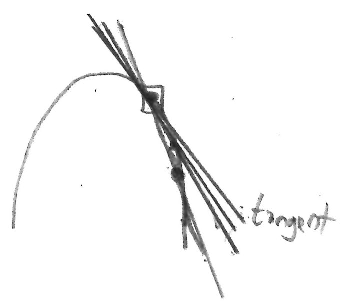
Exercise

1. Draw a tangent on the previous page at t = 7 seconds. Estimate the instantaneous rate of change at that point by calculating the slope of the tangent.

$$IROC = \frac{rise}{run}$$
$$= \frac{-100}{5}$$
$$= -20 m/s$$

2. The instantaneous rate of change at t = 7 seconds can be estimated by finding the average rate of change using the point t = 7 and a point very close to the right. Complete the table below and use it to estimate the instantaneous rate of change at t = 7 seconds.

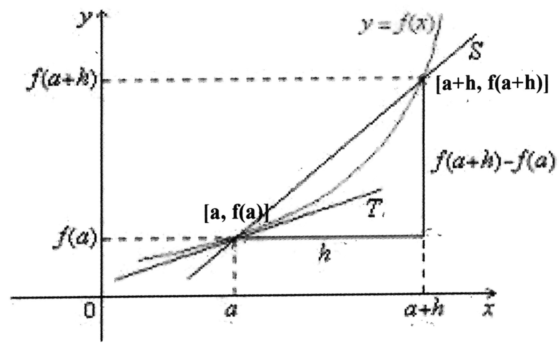
Interval	Average Rate of Change
$7 \le t \le 9$	$-30 m/s$
$7 \le t \le 8$	$-25 m/s$
$7 \le t \le 7.5$	$-22.5 m/s$
$7 \le t \le 7.25$	$-21.25 m/s$
$7 \le t \le 7.1$	$-20.5 m/s$
$7 \le t \le 7.01$	$-20.05 m/s$



$$AROC = \frac{H(9)-H(7)}{9-7}$$
$$= \frac{45-105}{9-7}$$
$$= -30 m/s$$

Approximation of IROC @ t=7 seconds

The instantaneous rate of change at the point where the independent variable equals 'a' can be estimated by calculating the average rate of change with a point that is 'h' units to the right; 'h' is a very small number.



To estimate the IROC, we use the **difference quotient**:

$$IROC \cong \frac{f(a+h)-f(a)}{(a+h)-a}$$

$$\boxed{IROC \cong \frac{f(a+h)-f(a)}{h}\text{, when h is “VERY” small}}$$

where

- 'a' is the value of the independent variable for which we would like to estimate the instantaneous rate of change.
- h is a "very small" interval size; set h = 0.01 for this course.

Example

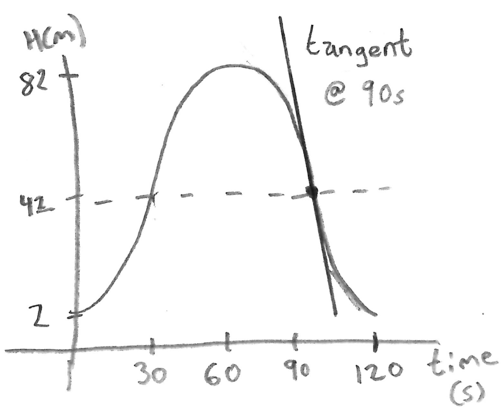
Justin Bieber boards a Ferris wheel to enjoy unique views of the metropolis. His height above ground is modeled by the equation: H(t) = - 40cos(3t) + 42 where

- H represents his height above the ground in metres.
- t represents the time in seconds.

$T = \frac{360°}{|k|} = \frac{360°}{|3|} = 120$

instantaneous

What is the rate of change after 90 seconds? Relate this answer to the scenario.



$$IROC \cong \frac{H(a+h)-H(a)}{h}$$

$$\cong \frac{H(90+0.01)-H(90)}{0.01}$$

$$\cong \frac{41.97905605-42}{0.01}$$

$\cong -2.09\,m/s$