Graphing Rational Functions: Part 1

rational function - a function that can be expressed as $f(x)=\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are each polynomial functions and $q(x)\neq 0$.

Example 1

Consider the function: $f(x)=\frac{\boxed{2}x+1}{\boxed{1}x-3}$.

a) Evaluate the following.

i) f(2.999)

$=-6998$

ii) f(3)

$=\frac{7}{0}$ ← non-zero, ← zero

$=$ undefined

iii) f(3.001)

$=7002$

b) What happens to this rational function when x = 3?

There is a vertical asymptote @ $x=3$

c) Evaluate the function at the end points $x \Rightarrow -\infty$ (approx. as -1,000,000) and $x \Rightarrow \infty$ (approx. as 1,000,000).

i) f(-1000000)

$=1.999993$

ii) f(1000000)

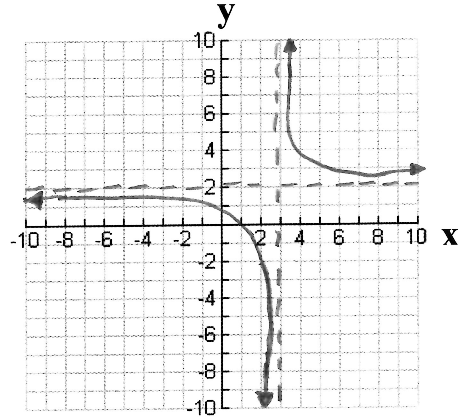
$=2.000007$

d) What value does the function approach at the end points and what does this suggest about the graph of f(x)?

The function approaches a value of 2 at the end points.

∴ There is a horizontal asymptote of $y=2$

e) Put the asymptotes on the graph below and sketch the graph of f(x). State the domain and range.



D: $\{x\in\mathbb{R}\mid x\neq 3\}$

R: $\{y\in\mathbb{R}\mid y\neq 2\}$

Example 2

Consider the function: $f(x) = \dfrac{x^2 - 6x + 8}{x - 4}$.

a) Complete the table of value below.

x	f(x)
1	-1
2	0
3	1
4	$\frac{0}{0}$ indeterminate $\left(\frac{\text{zero}}{\text{zero}}\right)$
5	3
6	4

b) Evaluate the following.

i) f(3.999)

$= 1.999$

ii) f(4)

$= \frac{0}{0}$ ← zero / ← zero

$=$ indeterminate

iii) f(4.001)

$= 2.001$

c) Factor and simplify the rational function. What happens when x = 4?

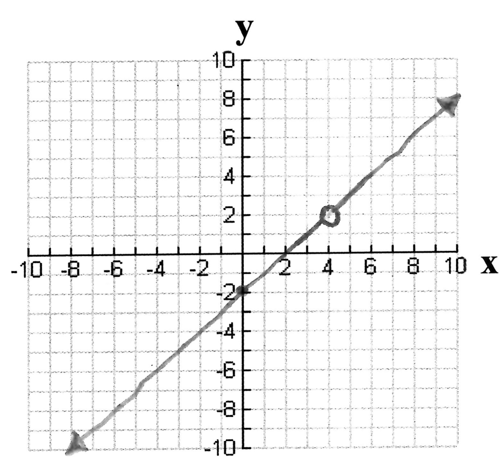
$f(x) = \dfrac{x^2 - 6x + 8}{x - 4}$

$= \dfrac{(x-2)\cancel{(x-4)}}{\cancel{(x-4)}}$

$= 1x - 2, \; x \neq 4$

∴ There is a hole @ $x = 4$

d) Graph the function and state the domain and range.



D: $\{x \in \mathbb{R} \mid x \neq 4\}$

R: $\{y \in \mathbb{R} \mid y \neq 2\}$

Example 3

Consider the function: $f(x) = \dfrac{x^2+4}{x+1}$.

a) i) f(-1.001)

$\cong -5002$

ii) f(-1)

$= \dfrac{5}{0}$ ← non-zero

= undefined

iii) f(-0.999)

$\cong 9998$

b) What happens to this rational function when x = -1?

It is undefined.

∴ There is a vertical asymptote @ x = -1.

c) Perform synthetic division for the rational function f(x); expect a remainder.

$$\begin{array}{r|rrr} -1 & 1 & 0 & 4 \\ & & -1 & 1 \\ \hline & 1 & -1 & 5 \end{array}$$

$y = 1x - 1$, R5

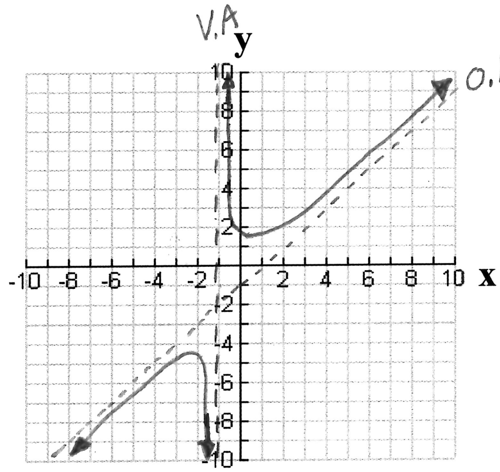
$y = x - 1 + \dfrac{5}{x+1}$

As $x \to \pm\infty$, this term approaches zero and essentially drops out. Near the end points, the graph will behave like $y = x - 1$. This is called an "oblique asymptote".

d) What does this result tell us about the graph of f(x)?

There is an oblique asymptote of $y = 1x - 1$

e) Graph the function.



Summary

Given $f(x)=\dfrac{p(x)}{q(x)}$

- If $f(a)=\dfrac{0}{0}$, then the factor 'x - a' can be factored out of $p(x)$ and $q(x)$ provided that we state that $x \neq a$. There is a hole at x = a.
- If $f(a)=\dfrac{non-zero\ number}{0}$, then the function has a vertical asymptote at x = a.
- If the degree of the numerator is less than the degree of the denominator, then $f(x)$ has a horizontal asymptote at y = 0.
- If the degree of the numerator = degree of the denominator, then f(x) has a horizontal asymptote as determined by the leading coefficients of $p(x)$ and $q(x)$.

  ex; $f(x)=\dfrac{\boxed{-12}x^3-2x-1}{\boxed{3}x^3+2x^2}$ --> $\rightarrow y=\dfrac{-12}{3}=-4$ H.A.
- If the degree of the numerator is greater than the degree of the denominator by exactly one, then f(x) has an oblique asymptote which is determined by dividing the dividend by the divisor and ignoring the remainder.

Homework: pg 262 # 1-3