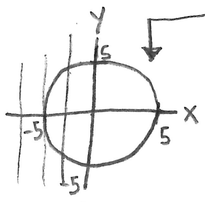
Grade 11 Review – Functions and their Notation

Function - a relation for which each value of the independent variable corresponds to at most ~~only~~ one value of the dependent variable. If x and y are two variables, then for each value of x, there is at most one value of y that corresponds with x. Another way of defining a function is to use a graph. For a function that is a relation, a vertical line will never cross more than one point on the plotted graph.

Example 1

Determine which if the following relationships are functions

$x^2+y^2=r^2$



a) $x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$\sqrt{y^2} = \pm\sqrt{25-x^2}$

$y = \pm\sqrt{25-x^2}$

when $x=3$, $y=\pm 4$

$\therefore$ Not a function

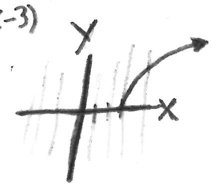
Fails V.L.T. (Vertical line test)

b) $y=\sqrt{2x-6}$

$y=\sqrt{2(x-3)}$

$k=2$

$d=3$

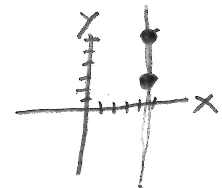


$\therefore$ It is a function.

c)

x	y
3	1
5	2
5	6
7	7
9	11

when $x=5$, $y=2,6$



$\therefore$ It is not a function.

Function Notation

The notation f(x) can be used to define any function. If the input variable x in f(x) is replaced by some value then the replacement must also occur for every other instance of x in the equation.

Example 2

1. Evaluate the following given $f(x) = x^2 - 3x + 1$

a) $f(4) = (4)^2-3(4)+1$

$= 16-12+1$

$= 5$

b) $f(-2) = (-2)^2-3(-2)+1$

$= 4+6+1$

$= 11$

The independent variable in a function is not restricted to only being replaced by a numerical value; it can also be replaced by an expression or even another function.

Example 3

2. Expand and simplify the following given $f(x) = 2x^2 + 1$ and $g(x) = 2x - 3$

a) $f(x-3) = 2(x-3)^2+1$

$= 2(x-3)(x-3)+1$

$= 2(x^2-6x+9)+1$

$= 2x^2-12x+18+1$

$= 2x^2-12x+19$

b) $g(x+5) = 2(x+5)-3$

$= 2x+10-3$

$= 2x+7$

c) $g(f(x)) = g(2x^2+1)$

$= 2(2x^2+1)-3$

$= 4x^2+2-3$

$= 4x^2-1$

Functions can be embedded or combined with other functions.

This equation is called the "difference quotient".

Instantaneous Rate of Change

Example 4

Simplify the following equations given f(x) = 5x + 7

a) y = 3$\boxed{f(x + 2)}$ – 8

$f(x+2) = 5(x+2)+7$
$= 5x+10+7$
$= \boxed{5x+17}$

$= 3(5x+17)-8$
$= 15x+51-8$
$y = 15x+43$

b) $IROC \cong \dfrac{f(x+h)-f(x)}{h}$

$\cong \dfrac{[5(x+h)+7]-[5x+7]}{h}$

$\cong \dfrac{\cancel{5x}+5h\cancel{+7}\cancel{-5x}\cancel{-7}}{h}$

$\cong \dfrac{5\cancel{h}}{\cancel{h}}$

$\cong 5,\ h \neq 0$

Equivalent functions can also be used to solve for a variable.

Example 5

If f(x) = 2x + 5, g(x) = x^2 – 3x - 5 and $\boxed{f(m + 3) = g(m + 1)}$ solve for m…

$(m+1)^2$
$= (m+1)(m+1)$
$= m^2+2m+1$

$f(m+3) = g(m+1)$
$2(m+3)+5 = (m+1)^2-3(m+1)-5$
$2m+6+5 = m^2+2m+1-3m-3-5$
$2m+11 = m^2-m-7$
$0 = m^2-3m-18$
$0 = (m-6)(m+3)$
$m = 6$ or $m = -3$

Functions can have multiple inputs but must only have one output.

Example 6 - Extension

If f(x, y, z) = 2xy + z^2 - x, evaluate the following:

a) f(1, 3, 5) $= 2(1)(3)+(5)^2-(1)$
$= 6+25-1$
$= 30$

b) f(-3, 2, -4) $= 2(-3)(2)+(-4)^2-(-3)$
$= -12+16+3$
$= 7$