Factored Form of Polynomials

Review: Factored Form of Quadratic Functions

1. a) Create a quadratic function that has x-intercepts of 3 and 7 and goes through the point (5,8).

$y = a(x - x_1)(x - x_2)$

$y = a(x-3)(x-7)$

sub in (5,8)

$8 = a(5-3)(5-7)$

$\frac{8}{-4} = \frac{-4a}{-4}$

$a = -2$

$\boxed{y = -2(x-3)(x-7)}$

b) Determine the y-intercept for the above function.

$y = -2(x-3)(x-7)$

set $x = 0$

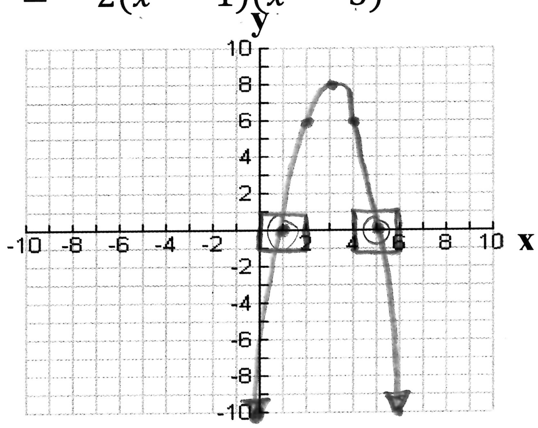
$y = -2(-3)(-7)$

y-int = −42

2. Graph the following quadratic functions expressed in factored form.

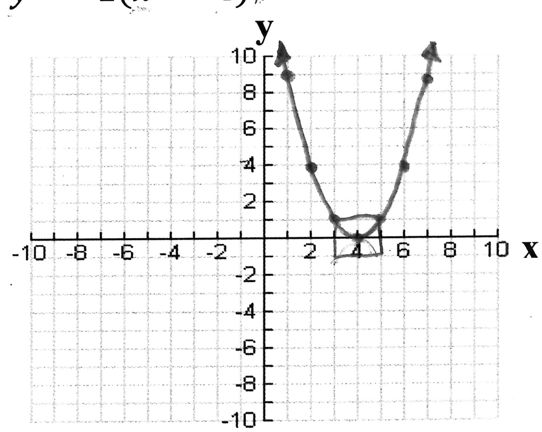
a) $y = -2(x-1)(x-5)$

x	y
0	-10
1	0
2	6
3	8
4	6
5	0
6	-10



b) $y = 1(x-4)^2$

x	y
1	9
2	4
3	1
4	0
5	1
6	4
7	9



3. a) List and describe the behaviour of the x-intercept(s) in graph a.

The x-intercepts are 1 and 5.
The graph behaves like a line crossing the x-axis at the x-ints; these are called "crossover x-intercepts".

b) List and describe the behaviour of the x-intercept(s) in graph b.

The x-intercept is 4.
The x-intercept behaves like a parabola near the x-axis around the x-intercept; this is called a turning point x-intercept.

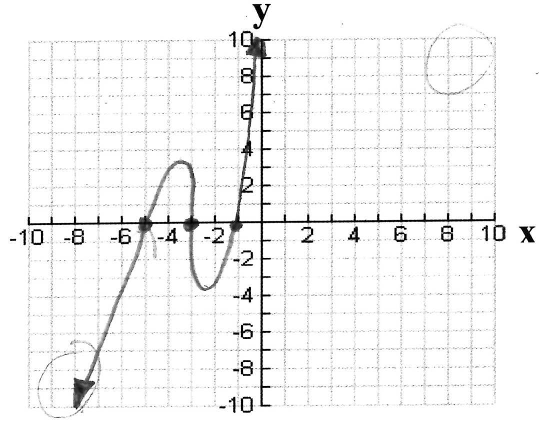
Sketching Polynomial Functions in Factored Form

1. Identify critical features of the function then create a sketch:

a) $f(x) = 2(x+1)^1(x+3)^1(x+5)^1$

lead. term $= 2x^3$

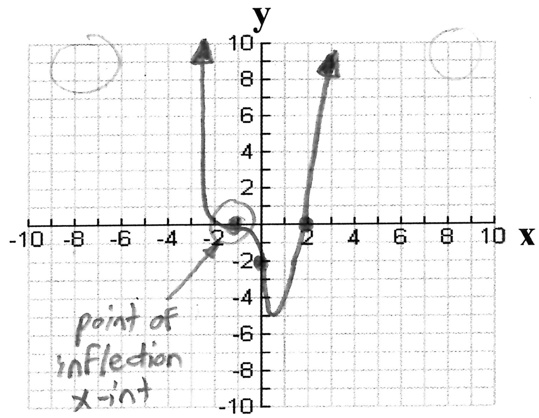
x-intercepts: $-1, -3, -5$
y-intercept: 30
leading coefficient: 2 (positive)
degree: 3 (odd)
end behaviours:
as $x \to -\infty$, $y \to -\infty$
as $x \to \infty$, $y \to \infty$

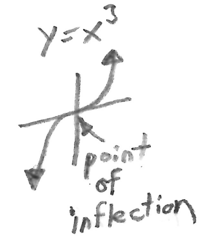


b) $f(x) = 1(x-2)^1(x+1)^3$

lead. term $= 1x^4$

x-intercepts: $2, -1$
y-intercept: -2
leading coefficient: 1 (positive)
degree: 4 (even)
end behaviours:
as $x \to -\infty$, $y \to \infty$
as $x \to \infty$, $y \to \infty$

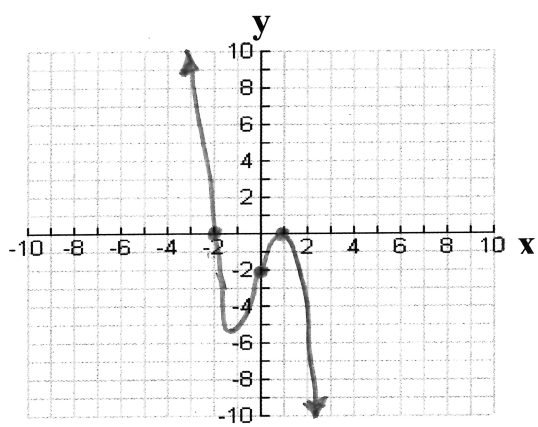




c) $f(x) = -1(x-1)^2(x+2)^1$

lead. term $= -1x^3$

x-intercepts: $1, -2$
y-intercept: -2
leading coefficient: -1 (negative)
degree: 3 (odd)
end behaviours:
as $x \to -\infty$, $y \to \infty$
as $x \to \infty$, $y \to -\infty$



Key Points pg 144–145

- End behaviours are determined by the degree of the function and the sign of the leading coeff.
- If a factor appears as (x - d), then the graph has an x-intercept at x = d and the graph is not tangent to the x-axis at that point; behaves like a line crossing x-axis.
- If a factor appears as $(x - d)^2$, then the graph has an x-intercept at x = d. This x-intercept is a turning point for the graph and is tangent to the x-axis at this point; these are local max/mins.
- If a factor appears as $(x - d)^3$, then the graph has an x-intercept at x = d. This x-intercept is a not a turning point for the graph but it is tangent to the x-axis at this point; point of inflection.

Determining the Equation of Polynomial Functions

Example 1

Determine the equation of a cubic function that has x-intercepts at 2, 4, and 6 and has a y-intercept of 6. Graph the function.

$y = a(x-x_1)(x-x_2)(x-x_3)$

$y = a(x-2)(x-4)(x-6)$

sub in (0,6)

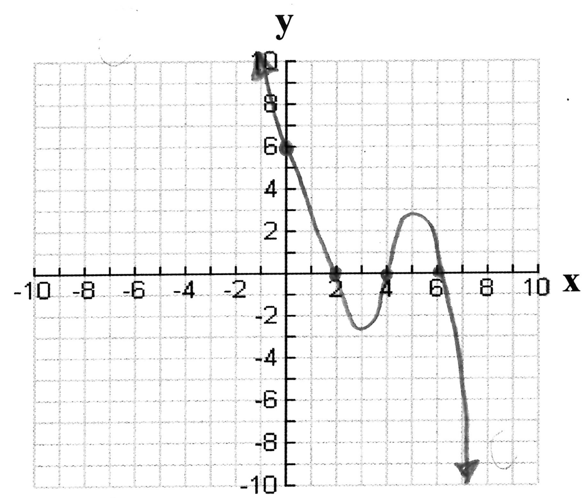
$6 = a(-2)(-4)(-6)$

$6 = -48a$

$\frac{6}{-48} = \frac{-48a}{-48}$

$a = -\frac{1}{8}$

$\boxed{y = -\frac{1}{8}(x-2)(x-4)(x-6)}$



lead term: $-\frac{1}{8}x^3$

lead coeff: $-\frac{1}{8}$ (neg)

degree: 3 (odd)

end behaviours:

as $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to -\infty$

Example 2

Determine the equation of a quartic function that has local maximums at the x-intercepts of 4 and 8 and passes through the point (6, -8). Graph the function.

turning point

$y = a(x-x_1)^2(x-x_2)^2$

$y = a(x-4)^2(x-8)^2$

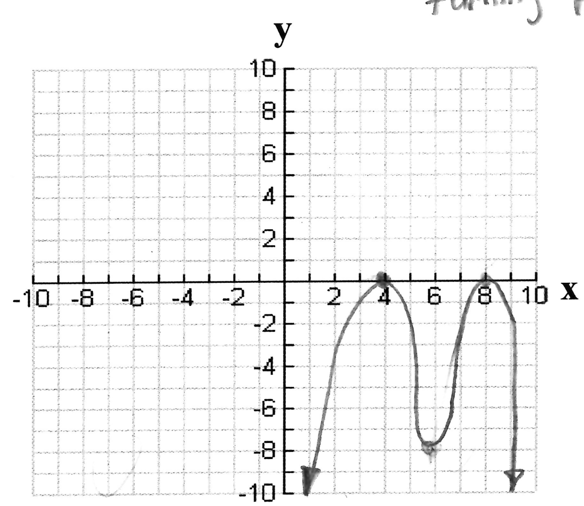
sub in (6,-8)

$-8 = a(6-4)^2(6-8)^2$

$\frac{-8}{16} = \frac{16a}{16}$

$a = -\frac{1}{2}$

$\boxed{y = -\frac{1}{2}(x-4)^2(x-8)^2}$



x-ints: 4, 8

y-int: -512

lead. term $= -\frac{1}{2}x^4$

lead. coeff. $= -\frac{1}{2}$ (neg.)

degree = 4 (even)

end behaviours:

as $x \to -\infty$, $y \to -\infty$

as $x \to \infty$, $y \to -\infty$

Hmwk: read pg 144-145, do pg 146 #1, 2, 3, 4, 6, 9ab, 10ac, 12, 13