

Review of Exponent Laws and Rational Exponents

Recall: Grade 9/10 Exponent Laws

Product Law: $a^m \cdot a^n = a^{m+n}$

Quotient Law: $\frac{a^m}{a^n} = a^{m-n}$

Power of a Power Law: $(a^m)^n = a^{mn}$

Power of a Product Law: $(ab)^m = a^m b^m$

Power of a Quotient Law: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Zero Exponent Law: $a^0 = 1$

Negative Exponent Law: $a^{-m} = \frac{1}{a^m}$ or $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

$(a+b)^n \neq a^n + b^n$

$(a-b)^n \neq a^n - b^n$

$\left(\frac{a}{1}\right)^{-m} = \left(\frac{1}{a}\right)^m = \frac{1^m}{a^m} = \frac{1}{a^m}$

Practice

Simplify and evaluate if possible:

a) $x^3 x^5 = x^8$

b) $(g^3)^2 = g^6$

c) $5.369^0 = 1$

d) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

e) $\frac{x^5}{x^2} = x^3$

f) $\frac{p^2}{p^6}$ or $p^{-4} = \frac{1}{p^4} = \frac{1}{p^6-2}$

g) $-3^2 = -9$

h) $(-3)^2 = (-3)(-3) = 9$

i) $(a^4 b^2)^3 = (a^4)^3 (b^2)^3 = a^{12} b^6$

j) $\left(\frac{2}{x}\right)^{-3} = \left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3}$

k) $\left(\frac{2x^2 y}{y^3}\right)^3 = \frac{(2^3)(x^2)^3(y)^3}{(y^3)^3} = \frac{8x^6 y^3}{y^9} = \frac{8x^6}{y^6}$

l) $\frac{4x^3 y^2 z^5}{8x^5 y^2} = \frac{yz^5}{2x^2}$ m) $-89^0 = -1$

$x^3 x^5 = (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x \cdot x) = x^8$

Recall: Rational Exponent Laws

Changing from Exponent to Root Form: $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$

Product of Roots: $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$

Quotient of Roots: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Practice

1. Simplify the following (express all exponents as positives):

a) $8^{\frac{2}{3}}$
 $= (8^{\frac{1}{3}})^2$ or $= (8^2)^{\frac{1}{3}}$
 $= (\sqrt[3]{8})^2$ $= \sqrt[3]{8^2}$
 $= (2)^2$ $= \sqrt[3]{64}$
 $= 4$ $= 4$

b) $\left(\frac{16}{81}\right)^{\frac{3}{4}}$
 $= \left(\frac{81}{16}\right)^{\frac{3}{4}}$
 $= \frac{81^{\frac{3}{4}}}{16^{\frac{3}{4}}}$
 $= \frac{(\sqrt[4]{81})^3}{(\sqrt[4]{16})^3}$
 $= \frac{(3)^3}{(2)^3}$
 $= \frac{27}{8}$

c) $x^{\frac{1}{3}}x^{\frac{1}{2}}$
 $= x^{\frac{1}{3} + \frac{1}{2}}$
 $= x^{\frac{2}{6} + \frac{3}{6}}$
 $= x^{\frac{5}{6}}$
 $= \frac{1}{x^{\frac{1}{6}}}$
 $= \frac{1}{\sqrt[6]{x}}$

d) $-4^{\frac{1}{2}}$
 $= -\sqrt{4}$
 $= -2$

e) $-2^{-\frac{1}{3} - \frac{4}{3}}$
 $= -2^{-\frac{5}{3}}$
 $= -2^0$
 $= -1$

f) $(4^{\frac{1}{2}})^2$
 $= 4^{\frac{1}{2} \cdot 2}$
 $= 4^1$
 $= 4$

2. Simplify the following expressions and express final answers in mixed root form. Simplified mixed radical form

a) $\frac{\sqrt{48}}{\sqrt{2}}$
 $= \sqrt{\frac{48}{2}}$
 $= \sqrt{24}$
 $= \sqrt{4} \sqrt{6}$
 $= 2\sqrt{6}$

b) $\sqrt{2}\sqrt{6}$
 $= \sqrt{12}$
 $= \sqrt{4}\sqrt{3}$
 $= 2\sqrt{3}$

c) $\frac{\sqrt{6}\sqrt{5}}{\sqrt{2}}$
 $= \frac{\sqrt{30}}{\sqrt{2}}$
 $= \sqrt{15}$

3. The value of a car is depreciating by 15% each year. If it is originally worth \$36,000, how much will it be worth in 5 years?

Exponential Depreciation

$y = a(1-r)^t$
 $= 36000(1-0.15)^t$
 $= 36000(0.85)^t$

Set $t=5$
 $y = 36000(0.85)^5$
 $y = \$15973.39$

Error on calculator
 \rightarrow
 $(-4)^{\frac{1}{2}}$