Hmwk: pg 127 # 1, 2, 3, 4, 5, 7, pg 122 # 1ad, 2, 3ab, 4a, 5, 7bc

Exploring Polynomial Functions

Polynomial Function - A function of the form
$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + \ldots + a_2 x^2 + a_1 x^1 + a_0$
such that

- $a_n, a_{n-1}, a_{n-2}, a_{n-3}, \ldots, a_2, a_1, a_0$ are all real numbers.
- n is a whole number; recall that whole numbers exclude negatives.

Polynomial functions are often written with the powers arranged in descending order. For example;

$$f(x) = 3x^2 + 5x - 6 \quad \text{(degree = 2)}$$
$$g(x) = 4x^5 - 3x^4 + 2x - 1 \quad \text{(degree = 5)}$$
$$h(x) = -3x^8 - 2x^3 + 5.8 \quad \text{(degree = 8)}$$
$$m(x) = -5x^3 + 3x^4 - 1 \quad \text{(degree = 4)}$$

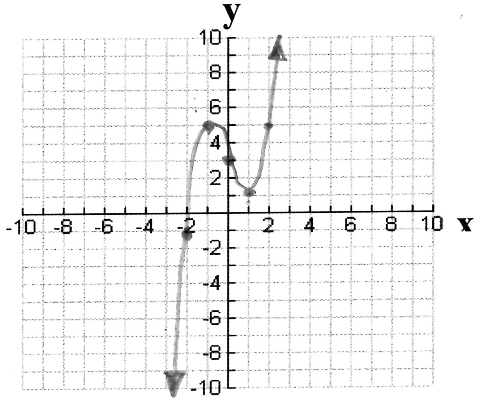
Should switch positions

Investigation

Graph the following polynomial functions and state the domain and range:

a) $f(x) = x^3 - 3x + 3$ (cubic)

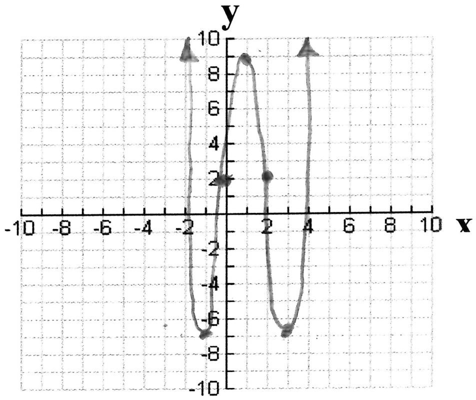
x	f(x)
-3	-15
-2	1
-1	5
0	3
1	1
2	5
3	21



Domain: $\{x \in \mathbb{R}\}$
Range : $\{y \in \mathbb{R}\}$

b) $f(x) = x^4 - 4x^3 - 2x^2 + 12x + 2$ (quartic)

x	f(x)
-3	137
-2	18
-1	-7
0	2
1	9
2	2
3	-7
4	18



Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y \geq -7\}$ *

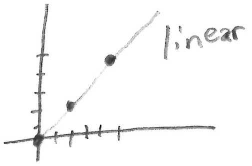
Properties of Polynomial Functions

- Polynomial functions can have an upper bound or a lower bound but not both.
- The graphs of polynomial functions do not have horizontal or vertical asymptotes.
- The y-intercept of a polynomial function is the constant term. For example; If $f(x) = x^3 - 2x^2 + 5x - 9$, the y-int is -9
- The domain of all polynomial functions $\{x \in \Re\}$.

Finite Differences

From a table of values, finite differences can be used to show that a polynomial function is of a certain degree. We already know that for linear relations, the first differences of the dependent variable are constant. For quadratic relations, the second differences are constant.

1st diff.	x	y	1st diff.
2	0	6	2
3	2	2	3
	5	5	



Practice

Use third and fourth differences to show that the relations in the previous question are cubic and quartic.

a)

1st diff	x	f(x)	1st diff.	2nd diff.	3rd diff.
1	-3	-15	16	-12	6
1	-2	1	4	-6	6
1	-1	5	-2	0	6
1	0	3	-2	6	6
1	1	1	4	12	
1	2	5	16		
	3	21			

The third diff. in y are constant.
∴ The relationship is cubic (degree = 3)

b)

1st diff.	x	f(x)	1st diff.	2nd diff,	3rd diff.	4th diff.
1	-3	137	-119	94	-60	24
1	-2	18	-25	34	-36	24
1	-1	-7	9	-2	-12	24
1	0	2	7	-14	12	24
1	1	9	-7	-2	36	
1	2	2	-9	34		
1	3	-7	25			
	4	18				

The fourth diff. in y are constant.
∴ The relationship is quartic (degree = 4)

if $n = 4$

$f(x) = a_4x^4 + a_3x^3 + \boxed{a_2x^2} + a_1x^1 + a_0$ (quartic)

ex; $f(x) = 5x^4 + 2x^3 - 4x^1 - 6$ $(a_2 = 0)$

if $n = 2$ (quadratic)

$f(x) = a_2x^2 + a_1x^1 + a_0$

$= ax^2 + bx + c$

$f(x) = 3x^{-2} + 1$

$= \frac{3}{1} \cdot \frac{1}{x^2} + \frac{x^2}{x^2}$

$= \frac{3}{x^2} + \frac{x^2}{x^2} \longrightarrow = \frac{3+x^2}{x^2}$ ← rational (not a polynomial)

$f(x) = 2x^{\frac{1}{2}} + 5$

$= 2\sqrt{x} + 5$ ← square root function (not a polynomial)