Hmwk: pg 466 # 1-6(ace), 9, 10, 11, 12, 15,18ace, 20

Evaluating Logarithms: Part 1

Recall:

- $\log_a x$ determines the exponent that needs to be placed on 'a' to equal 'x'.
- $y = a^x$ turns into $x = \log_a y$ in logarithmic form.

equivalent

Practice

Evaluate the following without a calculator:

$3^? = 81$

a) $\log_3 81$

$= 4$

$10^? = 1000$

b) $\log 1000$

base $= 10$

$= 3$

$10^? = -5$

c) $\log(-5)$

base $= 10$

$=$ No Sol^n

(Error)

d) $4^{\log_4 9}$

$4^? = 9$

$= 9$

$5^? = 5^{12}$

e) $\log_5(5^{12})$

$= 12$

f) $\log_7 \sqrt{7}$

$= \log_7 7^{\frac{1}{2}}$

$7^? = 7^{\frac{1}{2}}$

$= \frac{1}{2}$

Task: Prove the Change of Base (Splitting) Law.

Given $x = a^y$, show that $\log_a x = \dfrac{\log_b x}{\log_b a}$.

Method 1 (Write in Log Form)

$x = a^y$

① $y = \log_a x$

sub ① into ④

$$\boxed{\log_a x = \frac{\log_b x}{\log_b a}}$$

Method 2 (Change the Base)

Let $a = b^c$

$x = a^y$

$x = (b^c)^y$

$x = b^{cy}$

$\frac{cy}{c} = \frac{\log_b x}{c}$

② $y = \frac{\log_b x}{c}$

but $a = b^c$... so ③ $c = \log_b a$

sub ③ into ②

④ $y = \frac{\log_b x}{\log_b a}$

Example 1

The change of base law (or splitting) can be used to solve logarithmic equations. Use this law to solve the following:

a) $y = \log_3 9$

$= \frac{\log 9}{\log 3}$

$= 2$

b) $y = \log_2 9$

$2^? = 9$

$= \frac{\log 9}{\log 2}$

$\cong 3.17$

c) $y = \log_5 625$

$= \frac{\log 625}{\log 5}$

$= 4$

d) $2^x = 7$

$x = \log_2 7$

$= \frac{\log 7}{\log 2}$

$\cong 2.81$

e) $5^{x-2} = 100$

$x - 2 = \log_5 100$

$x = \frac{\log 100}{\log 5} + 2$

$x \cong 4.86$

f) $(2^x)^3 = 50$

$2^{3x} = 50$

$3x = \log_2 50$

$\frac{3x}{3} = \frac{\log 50}{\log 2} \div 3$

$x \cong 1.88$

Recall: Exponential Application Equations

Exponential Growth/Decay: $y = a(1 \pm r)^n$

Doubling Time: $y = a(2)^{\frac{t}{d}}$ ← doubling time

Half-Life: $y = a\left(\frac{1}{2}\right)^{\frac{t}{h}}$ ← half-life

Compound Interest: $A = P(1 + i)^n$

Example 2

The number of bacteria in a Petrie dish increases by a $\boxed{\text{factor of 5}}$ every 3 days. If there are originally 50 bacteria in the dish, how long will it take until there is 2000 bacteria?

$y = a(5)^{t/f}$ ← five fold time

$\frac{2000}{50} = \frac{50(5)^{\frac{t}{3}}}{50}$

$40 = 5^{t/3}$

$\frac{t}{3} = \log_5 40$

$\frac{t}{3} = \frac{\log 40}{\log 5}$

$\frac{t \log 5}{\log 5} = \frac{3 \log 40}{\log 5}$

$t \cong 6.88$ days