

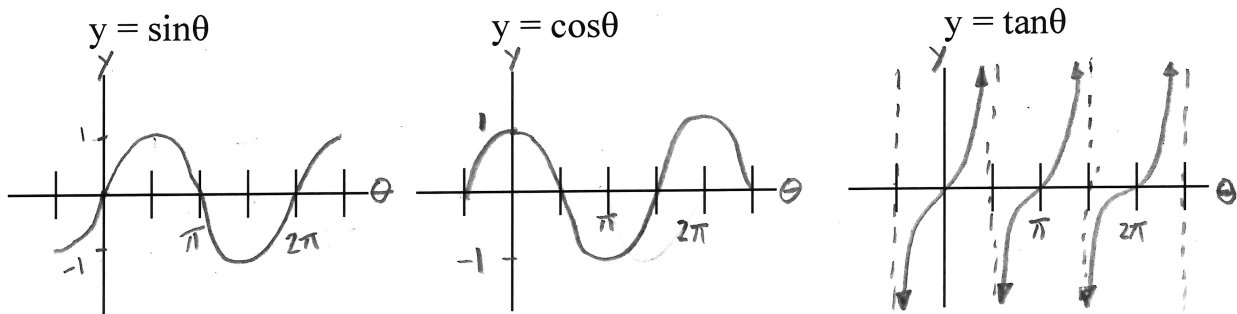
## Equivalent Trigonometric Functions

Some basic trigonometric identities can be created using three techniques:

1. Graph analysis
2. Trigonometric comparisons in a right triangle.
3. The CAST rule with related angles.

### Graph Analysis

Create sketches of each primary trigonometric functions;  $-\frac{\pi}{2} \leq \theta \leq \frac{5\pi}{2}$



- How can  $\sin\theta$  be expressed as a transformation of  $\cos\theta$ ?

$$\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$$

- How can  $\cos\theta$  be expressed as a transformation of  $\sin\theta$ ?

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$$

- The function  $y = \cos\theta$  is an even function. How can  $\cos\theta$  be expressed as a transformation of itself?

$$f(-x) = f(x) \quad \cos(-\theta) = \cos\theta$$

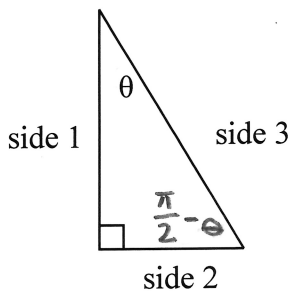
even

- The function  $y = \sin\theta$  and  $y = \tan\theta$  are odd functions. How can this be shown with transformations?

$$f(-x) = -f(x) \quad \left\{ \begin{array}{l} \sin(-\theta) = -\sin\theta \\ \tan(-\theta) = -\tan\theta \end{array} \right.$$

odd

## Using a Right Triangle (Cofunctions)

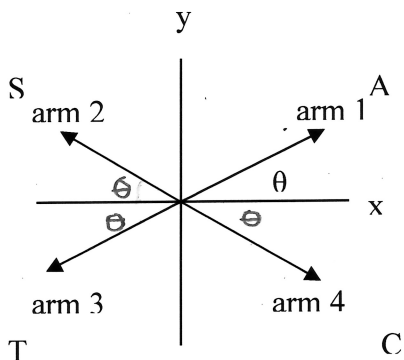


Label the angle in the lower right corner of the triangle above.

Determine two trigonometric expressions that are equivalent to each ratio.

Ratio	Equivalent Trigonometric Expressions
$\frac{\text{side 2}}{\text{side 3}}$	$\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$
$\frac{\text{side 1}}{\text{side 3}}$	$\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$
$\frac{\text{side 2}}{\text{side 1}}$	$\tan \theta = \cot\left(\frac{\pi}{2} - \theta\right)$

## The CAST Rule with Related Angles



In the above diagram the related angle of arm 1, arm 2, arm 3 and arm 4 are all equivalent. Determine the sine, cosine, and tangent for the terminal arm 2, arm 3, arm 4 in terms of the primary trigonometric values of arm 1.

	Arm 2	Arm 3	Arm 4
Sine	$\sin(\pi - \theta) = \sin \theta$	$\sin(\pi + \theta) = -\sin \theta$	$\sin(2\pi - \theta) = -\sin \theta$
Cosine	$\cos(\pi - \theta) = -\cos \theta$	$\cos(\pi + \theta) = -\cos \theta$	$\cos(2\pi - \theta) = \cos \theta$
Tangent	$\tan(\pi - \theta) = -\tan \theta$	$\tan(\pi + \theta) = \tan \theta$	$\tan(2\pi - \theta) = -\tan \theta$

$\tan(150^\circ)$   
  
 $= -\tan 30^\circ$   
 $= -\frac{\sqrt{3}}{3}$