Homework: pg 407 #1-3ace, 5, 7, 8, 9, 10, 11, 12a, 13ab, (17a)

Double Angle Formulas

Recall the compound angle formulas:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$
$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Also note that the Pythagorean Identity can be rearranged as follows:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\sin^2\theta = 1 - \cos^2\theta$$

Activity 1

The compound angle formulas can be used to create double angle formulas by expanding the following:

$\sin(a+b)$

a) $\sin(2\theta) = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\boxed{\sin(2\theta) = 2\sin\theta\cos\theta}$

b) $\cos(2\theta) = \cos(\theta + \theta)$

$(\sin\theta)^2 \Rightarrow \sin^2\theta$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$\boxed{\cos(2\theta) = \cos^2\theta - \sin^2\theta}$

$\cos(2\theta) = \cos^2\theta - (1 - \cos^2\theta)$

$\boxed{\cos(2\theta) = 2\cos^2\theta - 1}$

$\cos(2\theta) = (1 - \sin^2\theta) - \sin^2\theta$

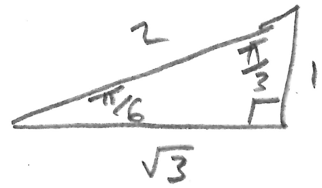
$\boxed{\cos(2\theta) = 1 - 2\sin^2\theta}$

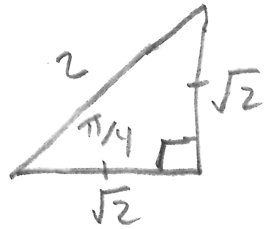
c) $\tan(2\theta) = \tan(\theta + \theta)$

$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$

$$\boxed{\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

Example 1





Simplify the following:

a) $8\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

$= 4 \cdot 2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

$= 4\sin\left[\frac{2}{1}\left(\frac{\pi}{8}\right)\right]$

$= 4\sin\left(\frac{\pi}{4}\right)$

$= \frac{4}{1}\frac{\sqrt{2}}{2}$

$= \frac{4\sqrt{2}}{2}$

$= 2\sqrt{2}$

b) $\dfrac{2\tan\left(\frac{\pi}{6}\right)}{1-\tan^2\left(\frac{\pi}{6}\right)}$

$= \tan\left[\frac{2}{1}\left(\frac{\pi}{6}\right)\right]$

$= \tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

Example 2

Use the double angle formulas to evaluate the following:

a) $\sin\left(\frac{\pi}{3}\right)$

$= \sin\left[2\left(\frac{\pi}{6}\right)\right]$

$= 2\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)$

$= \frac{\cancel{2}}{1}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2}$

b) $\cos\left(\frac{2\pi}{3}\right)$

$= \cos\left[2\left(\frac{\pi}{3}\right)\right]$

$= \cos^2\left(\frac{\pi}{3}\right) - \sin^2\left(\frac{\pi}{3}\right)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{4} - \frac{3}{4}$

$= -\frac{2}{4}$

$= -\frac{1}{2}$

Example 3

If $\cos\theta = \frac{3}{5}$ and $\pi \le \theta \le 2\pi$, determine the values of $\cos 2\theta$ and $\sin 2\theta$.

x

r

$x^2 + y^2 = r^2$

$(3)^2 + y^2 = (5)^2$

$9 + y^2 = 25$

$y^2 = 25 - 9$

$\sqrt{y^2} = \pm\sqrt{16}$

$y = \pm 4$ ← positive is inadmissable since $\pi \le \theta \le 2\pi$

$y = -4$

$\cos(2\theta) = 2\cos^2\theta - 1$

$= 2\left(\frac{3}{5}\right)^2 - 1$

$= \frac{2}{1}\left(\frac{9}{25}\right) - 1$

$= \frac{18}{25} - \frac{25}{25}$

$= -\frac{7}{25}$

$\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1}\left(\frac{-4}{5}\right)\left(\frac{3}{5}\right)$

$= -\frac{24}{25}$