Hmwk: pg 168 # 4, 6, 7ac, 8ac, 9ac, 10abde, 11, 12, 13, (17)

Dividing Polynomials: Part 2

$\frac{16}{7} = 2, R2$

$2 \times 7 + 2 = 16$

When polynomials are divided, the original dividend can be obtained by multiplying the quotient by the divisor then adding the remainder.

$$\frac{dividend}{divisor} = quotient, \quad remainder \qquad \text{or} \qquad (quotient \times divisor) + remainder = dividend$$

$\frac{\text{quot} \times \text{divisor}}{\text{quot}} = \frac{\text{dividend} - \text{remainder}}{\text{quot}}$

$\text{divisor} = \frac{\text{dividend} - \text{remainder}}{\text{quotient}}$

Practice

Divide the following pairs of polynomials then verify your answer.

a) $\frac{x^3 + 4x^2 - 26x + 15}{x - 3}$

$$\begin{array}{r} x^2 + 7x - 5, \ R0 \\ x-3 \overline{) x^3 + 4x^2 - 26x + 15} \\ x^3 - 3x^2 \\ \hline 7x^2 - 26x \\ 7x^2 - 21x \\ \hline -5x + 15 \\ -5x + 15 \\ \hline 0 \end{array}$$

Verify

quot. × divisor + remainder

$= (x^2+7x-5)(x-3) + 0$

$= x^3 - 3x^2 + 7x^2 - 21x - 5x + 15 + 0$

$= x^3 + 4x^2 - 26x + 15$

$=$ dividend ✓

b) $\frac{2x^3 + 7x^2 + 4x + 5}{2x + 3}$

$$\begin{array}{r} x^2 + 2x - 1, \ R8 \\ 2x+3 \overline{) 2x^3 + 7x^2 + 4x + 5} \\ 2x^3 + 3x^2 \\ \hline 4x^2 + 4x \\ 4x^2 + 6x \\ \hline -2x + 5 \\ -2x - 3 \\ \hline 8 \end{array}$$

verify

quot. × divisor + remainder

$= (x^2+2x-1)(2x+3) + 8$

$= 2x^3 + 3x^2 + 4x^2 + 6x - 2x - 3 + 8$

$= 2x^3 + 7x^2 + 4x + 5$ ✓

$=$ dividend

Synthetic Division

Synthetic division is a faster method for dividing polynomials by a binomial by using a series of additions and multiplications.

Example 1

Use synthetic division to simplify the following.

a) $\frac{x^3 + 4x^2 - 26x + 15}{x - 3}$

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -26 & 15 \\ & \downarrow & 3 & 21 & -15 \\ \hline & 1 & 7 & -5 & 0 \end{array}$$

0 ← remainder

$= 1x^2 + 7x - 5, \ R0$

b) $\frac{5x^3 - 7x^2 - x + 3}{x - 1}$

$$\begin{array}{r|rrrr} 1 & 5 & -7 & -1 & 3 \\ & \downarrow & 5 & -2 & -3 \\ \hline & 5 & -2 & -3 & 0 \end{array}$$

$= 5x^2 - 2x - 3, \ R0$

divisor

Synthetic division is often performed only with binomials in the form "x - a"; if there is a coefficient other than 1 in front of the x then it needs to be factored out of both terms first.

Consider the following:

$\frac{30}{15} \rightarrow \frac{30}{(5)(3)}$

The factors of the divisor 15 are 3 and 5. So, another way to perform this calculation above would be to write:

$30 \div 5 \div 3 = 2$

A similar approach can be used to divide polynomials by a binomial of the form "bx - a". For these cases, we first factor out the constant b. Then we divide by the new binomial and finish by dividing this answer by the constant b.

<u>Example 2</u>

Use synthetic division to simplify the following.

$-\frac{3}{2} \cdot \frac{2}{1} = -\frac{6}{2} = -3$

a) $\frac{2x^4 - 20x^2 + 26x - 60}{2x - 6}$ missing "x^3" term

$2(x-3)$

$$\begin{array}{r|rrrrr} 3 & 2 & 0 & -20 & 26 & -60 \\ & \downarrow & 6 & 18 & -6 & 60 \\ \hline & 2 & 6 & -2 & 20 & 0 \end{array}$$

$= \frac{2x^3+6x^2-2x+20}{2}$, R0

$= x^3+3x^2-x+10$, R0

b) $\frac{2x^3 - x^2 + 4x + 15}{2x + 3}$

$2(x+\frac{3}{2})$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & 4 & 15 \\ & & -3 & 6 & -15 \\ \hline & 2 & -4 & 10 & 0 \end{array}$$

$= \frac{2x^2-4x+10}{2}$, R0

$= x^2-2x+5$, R0

c) $\frac{2x^3 - x^2 - 20}{2x - 5}$ missing 'x' term

$2(x-\frac{5}{2})$

$$\begin{array}{r|rrrr} \frac{5}{2} & 2 & -1 & 0 & -20 \\ & & 5 & 10 & 25 \\ \hline & 2 & 4 & 10 & 5 \end{array}$$

$= \frac{2x^2+4x+10}{2}$, R5

$= x^2+2x+5$, R5

* Note: Do not divide the remainder by 2.