Hmwk: pg 168 # 1-3, 5

Dividing Polynomials: Part 1

Investigation

1. Expand the following products.

a) (6)(3)

$= 18$

b) $(x - 3)(x + 5)$

$= x^2 + 5x - 3x - 15$

$= x^2 + 2x - 15$

c) $(x^2 + 4x - 1)(x - 1)$

$= x^3 - x^2 + 4x^2 - 4x - 1x + 1$

$= x^3 + 3x^2 - 5x + 1$

To divide an expression by another, the answer can be found by determining what needs to be multiplied by the divisor to get the dividend. For example, suppose we are asked to divide the following:

$$\frac{18}{3}$$

This question can be reworded to, "What needs to be multiplied by 3 to get 18?"

ie; $? \times 3 = 18$

The same thought process can be used to divide polynomials.

When asked to divide...

$$\frac{x^3 + 3x^2 - 5x + 1}{x - 1}$$

We are being asked, what must be multiplied by 'x - 1' to result in '$x^3 + 3x^2 - 5x + 1$':

ie; $? \bullet (x - 1) = x^3 + 3x^2 - 5x + 1$

The work in question 1c) suggests that the answer should be:

$$x^2 + 4x - 1$$

The Long Division Method

Use long division to divide the following polynomials.

$(3x^2) - (-1x^2) = 4x^2$

a) $\dfrac{x^3+3x^2-5x+1}{x-1}$

$x^2+4x-1,\ R0$

$x-1\ \overline{)\,x^3+3x^2-5x+1}$

x^3-1x^2

$4x^2-5x$

$4x^2-4x$

$-1x+1$

$-1x+1$

0

b) $\dfrac{x^2+2x-15}{x-3}$

$x+5,\ R0$

$x-3\ \overline{)\,x^2+2x-15}$

x^2-3x

$5x-15$

$5x-15$

0 ← remainder

$\dfrac{x^2+2x-15}{x-3}$

$= \dfrac{(x+5)\cancel{(x-3)}}{\cancel{(x-3)}}$

$= x+5,\ x \neq 3$

c) $\dfrac{-5x^2+x^3+7x+3}{x-2}$

$x^2-3x+1,\ R5$

$x-2\ \overline{)\,x^3-5x^2+7x+3}$

x^3-2x^2

$-3x^2+7x$

$-3x^2+6x$

$x+3$

$x-2$

5 ← non-zero remainder

$\dfrac{16}{5} = 3, R1$

d) $\dfrac{x^4+2x^3+5x^2-36}{x^2-2}$

missing the 'x' term → requires a "filler" (0x)

$x^2+2x+7,\ R\ 4x-22$

$x^2+0x-2\ \overline{)\,x^4+2x^3+5x^2+0x-36}$

$x^4+0x^3-2x^2$

$2x^3+7x^2+0x$

$2x^3+0x^2-4x$

$7x^2+4x-36$

$7x^2+0x-14$

$4x-22$

$\boxed{\dfrac{\text{dividend}}{\text{divisor}} = \text{quotient, remainder}}$

Note:

- When using long division, the divisor and dividend should be written out in descending orders of power.
- Zero should be used as the coefficient of any power missing in the divisor and/or the dividend.
- ★ If the remainder is zero, then both the divisor and quotient are factors of the dividend.
- A polynomial can only be divided by a polynomial of less or equal degree.