Name: Sol^n

MHF4U – Diagnostic Assessment

Use scrap paper to solve each question then place final answers in the spaces provided.

1. Given the function $f(x) = x^2 - 3x + 5$, expand and simplify the following:

a) $f(-3) - f(2)$ — 20

b) $f(x - 1)$ — $x^2 - 5x + 9$

c) $\frac{f(x+h)-f(x)}{h}$ — $2x + h - 3$, $h \neq 0$

d) $3f(x - 1) + 5$ — $3x^2 - 15x + 32$

2. If $f(x) = 2x + 1$ and $g(x) = 3x^2 - 6$, expand and simplify $f(g(x))$ — $6x^2 - 11$

3. Fully factor $6x^2 + 11x - 10$ — $(3x-2)(2x+5)$

4. Fully factor $-2x^2 - 4x + 10$ — $-2(x^2+2x-5)$

5. For each rational function below, state the implicit restrictions:

a) $\frac{x}{3} + \frac{y}{2x-3}$ — $x \neq 3/2$

b) $\frac{x+2}{4} \times \frac{3}{x-y}$ — $x \neq y$

c) $\frac{y}{5} \div \frac{x+2}{3}$ — $x \neq -2$

6. Simplify each rational expression; do not state the restrictions:

a) $\frac{2}{x+3} - \frac{5}{x^2+2x-3}$ — $\frac{2x-7}{(x+3)(x-1)}$

b) $\frac{2x+10}{x-3} \times \frac{x+1}{x^2+6x+5}$ — $\frac{2}{x-3}$

c) $\frac{2}{x-1} + \frac{3}{x}$ — $\frac{5x-3}{x(x-1)}$

7. How many real solutions does the equation $x^2 = 5x - 7$ have? $\emptyset$

8. Determine the equation of a quadratic function that has x-intercepts of 3 and 7 and goes through the point (8, -10) $y = -2(x-3)(x-7)$

9. Determine the coordinates of the vertex for the quadratic function $y = 2x^2+4x-6$ $(-1, -8)$

10. Solve the following exponential equations:

a) $3^x = 30$ 3.1

b) $4^{3x+1} = 32^{x+1}$ 3

c) $2^{x+3} = -16$ No Sol^n

11. Evaluate the following using exact values:

a) sin(60°) $\frac{\sqrt{3}}{2}$

b) cos(135°) $-\frac{\sqrt{2}}{2}$

c) tan(-150°) $\frac{\sqrt{3}}{3}$

12. If the coordinates of a point on the end of a terminal arm is (12, -5), what is the angle in standard position? $337.4°$

13. Solve the following trigonometric equations given $0^o \leq \theta \leq 360^o$:

a) $cos\theta = -0.5$ $120°$ or $240°$

b) $sin\theta = 1$ $90°$

c) $cot\theta = \frac{1}{\sqrt{3}}$ $60°$ or $240°$

14. List features for the sinusoidal function $y = -3cos(4\theta - 120^o) + 4$

a) Amplitude 3

b) Period $90°$

c) Range $\{y \in \mathbb{R} \mid 1 \leq y \leq 7\}$

15. Graph the following

a) $y = 2\sqrt{-x+1} - 4$

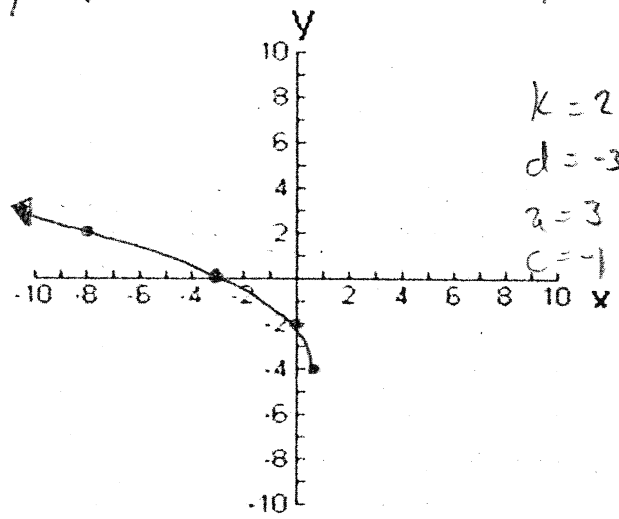
$y = 2\sqrt{-(x-1)} - 4$

$k = -1$

$d = 1$

$a = 2$

$c = -4$



b) $y = 3\cos(2\theta + 60°) - 1$

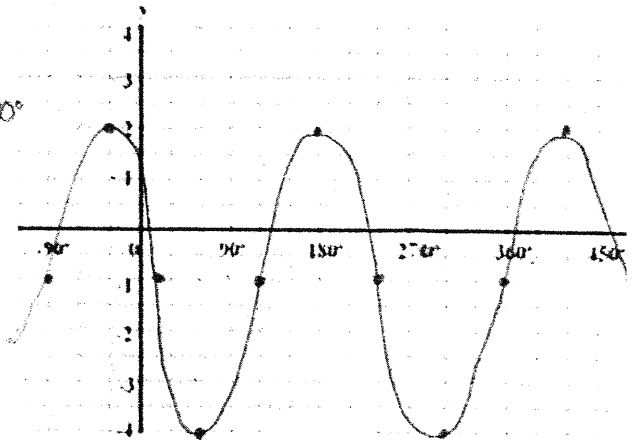
$y = 3\cos[2(\theta + 30°)] - 1$

$k = 2$

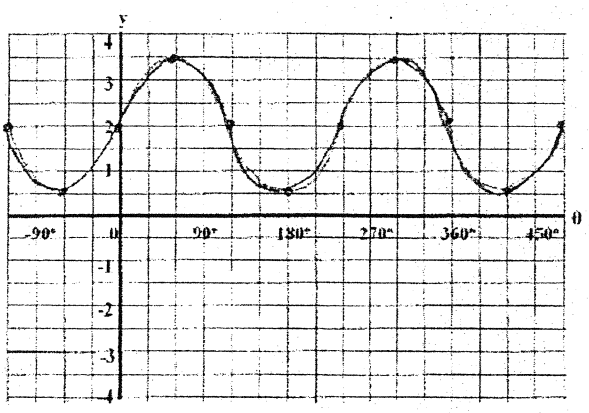
$d = -30°$

$a = 3$

$c = -1$



16. Consider the graph for the sinusoidal function $y = 1.5\cos[k(\theta - d)] + 2$ below



What is the value of k? 1.5

What is the smallest positive value of d? 60°

17. Determine the inverse of the function $y = -\sqrt{x-2}$ $y = x^2 + 2, x \leq$

18. Is the inverse of $y = 4x^2 - 18$ a function? No

Diagnostic Solutions

$$f(x) = x^2 - 3x + 5$$

1a) $f(-3) - f(2)$

$= [(-3)^2 - 3(-3) + 5] - [(2)^2 - 3(2) + 5]$

$= [9 + 9 + 5] - [4 - 6 + 5]$

$= [23] - [3]$

$= 20$

b) $f(x-1) = (x-1)^2 - 3(x-1) + 5$

$= (x-1)(x-1) - 3x + 3 + 5$

$= x^2 - 2x + 1 - 3x + 3 + 5$

$= x^2 - 5x + 9$

c) $\dfrac{f(x+h) - f(x)}{h} =$

$= \dfrac{[(x+h)^2 - 3(x+h) + 5] - [x^2 - 3x + 5]}{h}$

$= \dfrac{[x^2 + 2xh + h^2 - 3x - 3h + 5] - [x^2 - 3x + 5]}{h}$

$= \dfrac{x^2 + 2xh + h^2 - \cancel{3x} - 3h + \cancel{5} - x^2 + \cancel{3x} - \cancel{5}}{h}$

$= \dfrac{2xh + h^2 - 3h}{h}$

$= \dfrac{\cancel{h}(2x + h - 3)}{\cancel{h}}$

$= 2x + h - 3, \quad h \neq 0$

d) $3\boxed{f(x-1)} + 5$

$= 3(x^2 - 5x + 9) + 5$

$= 3x^2 - 15x + 27 + 5$

$= 3x^2 - 15x + 32$

$f(x-1) = (x-1)^2 - 3(x-1) + 5$

$= x^2 - 2x + 1 - 3x + 3 + 5$

$= x^2 - 5x + 9$

2. $f(x) = 2x+1$ $\quad g(x) = 3x^2-6$

$f(g(x))$

$= F(3x^2-6)$

$= 2(3x^2-6)+1$

$= 6x^2-12+1$

$= 6x^2-11$

3. $6x^2+11x-10$ $\quad P(-60)\}\ 15,-4$ $\quad S(11)$

$= 6x^2+15x-4x-10$

$= 3x(2x+5)-2(2x+5)$

$=(3x-2)(2x+5)$

4. $-2x^2-4x+10$

$=-2(x^2+2x-5)$

5. a) $\frac{x}{3} + \frac{y}{2x-3}$ $\quad 2x-3 \neq 0$, $\frac{2x}{2} \neq \frac{3}{2}$, $x \neq \frac{3}{2}$

b) $\frac{x+2}{4} \times \frac{3}{x-y}$ $\quad x-y \neq 0$, $x \neq y$

c) $\frac{y}{5} \div \frac{x+2}{3}$ $\quad x+2 \neq 0$, $x \neq -2$

6. a) $\frac{2}{x+3} - \frac{5}{x^2+2x-3}$

$= \frac{(x-1)}{(x-1)}\frac{2}{(x+3)} - \frac{5}{(x+3)(x-1)}$

$= \frac{2x-2}{(x-1)(x+3)} - \frac{5}{(x+3)(x-1)}$

$= \frac{2x-7}{(x-1)(x+3)}$

b) $\frac{2x+10}{x-3} \times \frac{x+1}{x^2+6x+5}$

$= \frac{2(x+5)(x+1)}{(x-3)(x+5)(x+1)}$

$= \frac{2}{x-3}$

c) $\frac{2(x)}{(x-1)(x)} + \frac{3(x-1)}{(x)(x-1)}$

$= \frac{2x}{(x-1)x} + \frac{3x-3}{x(x-1)}$

$= \frac{5x-3}{x(x-1)}$

7. $x^2 = 5x - 7$

$x^2 - 5x + 7 = 0$

discriminant $\rightarrow b^2 - 4ac$

$= (-5)^2 - 4(1)(7)$

$= 25 - 28$

$= -3$

< 0 $\therefore$ No real sol^{ns}

8. $y = a(x - x_1)(x - x_2)$

$y = a(x - 3)(x - 7)$

sub in $(8, -10)$

$-10 = a(8-3)(8-7)$

$\frac{-10}{5} = \frac{5a}{5}$

$a = -2$

$y = -2(x-3)(x-7)$

9. $y = 2x^2 + 4x - 6$

$y = 2(x^2 + 2x) - 6$

$\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$

$y = 2(x^2 + 2x + 1 - 1) - 6$

$y = 2(x^2 + 2x + 1) - 2 - 6$

$y = 2(x+1)^2 - 8$

vertex $(-1, -8)$

or

$y = 2x^2 + 4x - 6$

$y = 2(x^2 + 2x - 3)$

$y = 2(x+3)(x-1)$

x-ints: -3 & 1

vertex

$x = \frac{-3+1}{2}$ $\quad y = 2(-1)^2 + 4(-1) - 6$

$x = -1$ $\quad y = 2 - 4 - 6$

$y = -8$

10a) $3^x = 30$

$\log 3^x = \log 30$

$\frac{x \log 3}{\log 3} = \frac{\log 30}{\log 3}$

$x \approx 3.1$

b) $4^{3x+1} = 32^{x+1}$

$(2^2)^{3x+1} = (2^5)^{x+1}$

$2^{6x+2} = 2^{5x+5}$

$6x + 2 = 5x + 5$

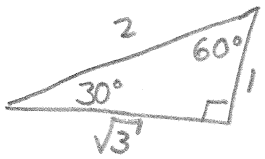
$6x - 5x = 5 - 2$

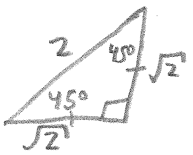
$x = 3$

c) $2^{x+3} = -16$

No exponent can be placed on '2' to make it negative.

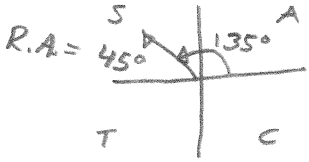
$\therefore$ No solution.





11. a) $\sin(60°) = \frac{\sqrt{3}}{2}$

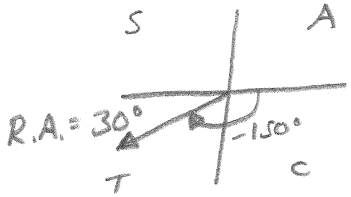
b) $\cos(135°)$



$= -\cos(45°)$

$= -\frac{\sqrt{2}}{2}$

c) $\tan(-150°)$

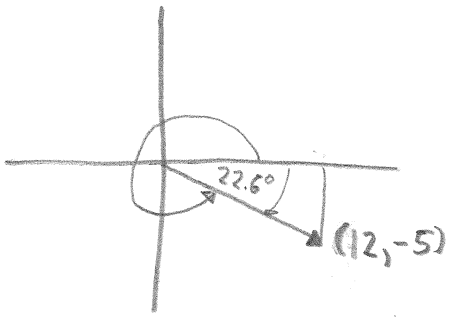


$= \tan(30°)$

$= \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

12. a)



$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-5}{12}$

$\theta = \tan^{-1}\left(\frac{-5}{12}\right)$

$\theta = -22.6°$

but $0° \le \theta \le 360°$

so...

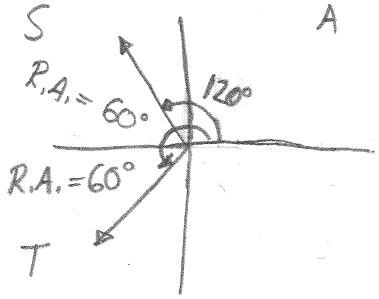
$\theta = 360° - 22.6°$

$\cong 337.4°$

13 a) $\cos\theta = -0.5$

$\theta = \cos^{-1}(-0.5)$

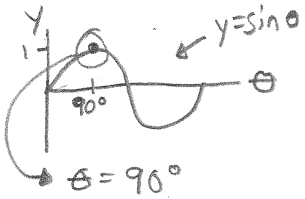
$\theta_1 = 120°$



$\theta_2 = 180° + 60°$

$\theta_2 = 240°$

b) $\sin\theta = 1$



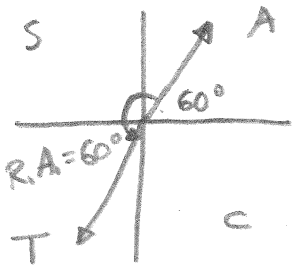
$\theta = 90°$

c) $\cot\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta_1 = 60°$



$\theta_2 = 180° + 60°$

$\theta_2 = 240°$

14. $y = -3\cos(4\theta - 120°) + 4$

a) amplitude $= |a| = |-3| = 3$

b) period $= \frac{360°}{k} = \frac{360°}{4} = 90°$

c) line of equilibrium $y = 4$

amplitude $= 3$

maximum $=$ line of equilibrium $+$ amplitude

$= 4 + 3$

$= 7$

minimum $=$ line of equilibrium $-$ amplitude

$= 4 - 3$

$= 1$

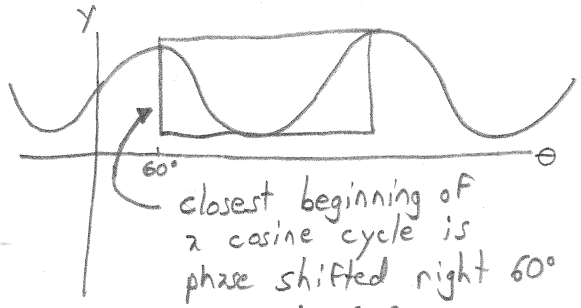
Range: $\{ y \in \mathbb{R} \mid 1 \leq y \leq 7 \}$

15. See graph on answer set.

16. $k = \frac{360°}{T}$

$= \frac{360°}{240°}$

$= 1.5$



closest beginning of a cosine cycle is phase shifted right 60°

so,.. $d = 60°$

17. $y = -\sqrt{x-2}$

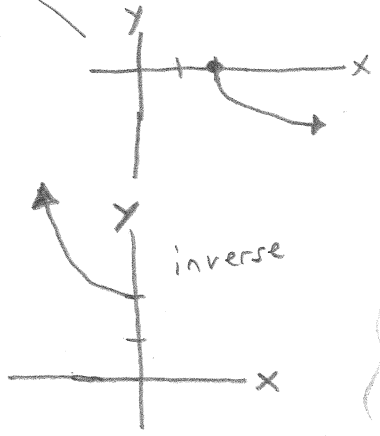
$x = -\sqrt{y-2}$

$(-\sqrt{y-2})^2 = (x)^2$

$(-1)^2(\sqrt{y-2})^2 = x^2$

$y - 2 = x^2$

$y = x^2 + 2, \quad x \leq 0$



18. $y = 4x^2 - 18$